Harvest and Sowing

Reflection and testimony
on a past as a mathematician

by

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Presentation of Themes:

or

PRELUDE TO FOUR MOVEMENTS

Fascicle 0:

By way of a Preface
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Introduction

To my Parents

N.B. The present "fascicule 0_1" of the provisional edition of Harvest and Sowing is intended (as the table of contents shows) to be placed before the fascicule (taking the place of no. 0_2) which had been distributed previously, under the title "Letter - Introduction"; with the exception, however, of the "Epilogue in postscript" (numbered from L 44 to L 56), which constitutes (as its name indicates) a "post-scriptum" to the "Letter" (pages L 1 to L 43) opening the "fascicule 0_2". Together, the two fascicles constitute the introductory part of Harvest and Sowing, called “Presentation of Themes" or "Prelude to Four Movements".

As a Forward...

30 January 1986

Only the foreword remains to be written before sending Harvest and Sowing to the printer. And I swear I had the best will in the world to write something that would do the job. Something <u>reasonable</u> this time. Three or four pages, no more, but well felt, to present this enormous "pavé" of more than a thousand pages. Something that will "hook" the jaded reader, which will make him see that in these not very reassuring "more than a thousand pages", there could be things that interest him (or even concern him, who knows?). It's not so much my style, the hook, it's not. But here I am going to make an exception, for once! "The publisher who is crazy enough to take the risk" (of publishing this obviously unpublishable monster) has to cover the costs somehow.

And then no, this didn't happen. I did my best though. And not just for one afternoon, as I had intended, quickly. Tomorrow I'll have been working on it for exactly three weeks and the sheets have been piling up. The result, for sure, is not what one could decently call a "foreword". It's failed again, definitely! One can't do it again at my age - and I'm not made for sale or be sold. Even when I want to please people (myself and my friends...).

What has resulted is a sort of long commented "promenade" through my work as a mathematician. A promenade intended above all for the 'layman' - for those who 'have never understood anything about maths'. And also for me, who had never taken a leisure of such a promenade. One thing leading to another, I was led to bring out and say things that until then had always remained unspoken. As if by chance, these are also the things that I feel are the most essential in my work and in my oeuvre. These are things that are not at all technical. It's up to you to see if I have succeeded in my naive attempt to "get them across" - a somewhat crazy undertaking as well, for surely. My satisfaction and my pleasure would be to have been able to make you feel them. Things that many of my learned colleagues can no longer feel. Perhaps they have become too learned and too prestigious. This makes you lose touch, very often, with the simple and essential things.

During this "promenade through a work", I also talk a little about my life. And a little bit, here and there, about what Harvest and Sowing is about. I will talk about it again, and in more detail, in the "Letter" (dated May last year) which follows the "Promenade". This letter was intended for my ex-students and my "friends of yesteryear" in the mathematical world. But there is nothing technical about it either. It can be read without any problem by any reader who would be interested in learning, through an "on the spot" account, the ins and outs that finally led me to write Harvest and Sowing. Even more than the Promenade, it will also give you a taste of a certain atmosphere, in the "big" mathematical world. And also (just like the Promenade), of my style of expression, which is a bit special, apparently. And also the spirit that is expressed in this style - a spirit that is not appreciated by everyone either.

In the Promenade and throughout Harvest and Sowing, I talk about the <u>mathematical work</u>. It is the work that I know well and first-hand. Most of what I say about it is true, surely, for all creative work, all work of discovery. It is true at least for the so-called "intellectual" work, that which is done mainly "in the head", and by writing. Such work is marked by the blossoming and development of an <u>understanding</u> of the things we are probing.

But, to take an example at the opposite end of the spectrum, the passion of love is also a drive for discovery. It opens us up to a so-called 'carnal' knowledge, which also renews itself, blossoms and deepens. These two impulses - the one that drives the mathematician at work, let's say, and the one in the lover - are much closer than we generally suspect, or are willing to admit to ourselves. I hope that the pages of Harvest and Sowing will help you feel this, in your work as well as your everyday life.

In the course of the Promenade, the mathematical work itself will be discussed. However, I am almost silent on the <u>context</u> in which this work takes place, and on the <u>motivations</u> that come into play outside of the actual working time. This risks in giving my person, or the mathematician or the "scientist" in general, a flattering but distorted image. Like "great and noble passion", without any correction. In line, in short, with the great "Myth of Science" (with a capital S if you like!). The heroic, "Promethean" myth into which writers and savants have fallen (and continue to fall) one after another*[à qui mieux mieux]*. Only historians, perhaps, sometimes resist this seductive myth. The truth, is that in the motivations of 'the scientist', which sometimes push him to invest lavishly in his work, ambition and vanity play as important and almost universal a role as in any other profession. It takes more or less crude, more or less subtle forms, depending on the person concerned. I do not claim to be an exception. Reading my testimony will, I hope, leave no doubt about this.

It is also true that the most devouring ambition is powerless to discover the slightest mathematical statement, or to prove it - just as it is powerless (for example) to "turn it on" (in the true sense of the word). Be it a woman or a man, what "turns it on" is not ambition, the desire to shine, to show off a power, sexual in this case - quite the contrary! But it is the acute perception of something strong, very real and very delicate at the same time. It can be called "beauty", and that is one of the thousand faces of this thing. Being ambitious does not necessarily prevent one from feeling the beauty of a being, or of a thing, that's true. But what is certain, is that it is not ambition that enables us feel it...

The man who first discovered and controlled the fire was someone exactly like you and me. Not at all what we think of as a "hero", "demigod" and so on. Surely, like you and me, he knew the sting of the agony, and the proven ointment of vanity, which makes one forget the sting. But at the moment he "knew" the fire, there was no fear, no vanity. Such is the truth in the heroic myth. The myth becomes insipid, it becomes ointment, when it serves to hide from us <u>another</u> aspect of things, which is just as real and essential.

My purpose in Harvest and Sowing is to talk about both aspects - about the drive to know, and about fear and its antidotes of vanity. I think I 'understand', or at least <u>know</u> the drive and its nature. (Perhaps one day I will discover, in wonder, how much I was deluding myself...) But as for fear and vanity, and the insidious blockages to creativity that derive from them, I know full well that I have not got to the bottom of this great enigma. And I don't know if I'll ever get to the bottom of that mystery, in the years I have left to live...

In the course of writing Harvest and Sowing two images emerged, to represent both of these two aspects of the human adventure. They are the <u>child</u> (alias the <u>worker</u>), and the <u>Boss</u>. In the Promenade we are about to take, it is the "child" that will be discussed almost exclusively. He is also the one who appears in the subtitle "<u>The Child and the Mother</u>". This name will become clearer, I hope, in the course of the promenade.

In all the rest of the reflection, however, it is the Boss who takes centre stage. He is not a boss for nothing! It would be more accurate to say that it is not one Boss, but Bosses of competing companies. But it is also true that all the Bosses are alike on the essential. And when we start talking about the Bosses, it also means that there will be the "villains". In Part I of the reflection ("Fatuity and Renewal", which follows this introductory part, or the "Prelude to Four Movements"), it is mostly me, "the villain". In the following three parts, it is mostly "the others". Each in his own turn!

This means that, in addition to deep philosophical reflections and (by no means contrite) "confessions", there will be "vitriolic portraits" (to use the expression of one of my colleagues and friends, who has found himself a bit manhandled). Not to mention the large-scale "operations" that were quite of a kind[pas piquées de vers]. Robert Jaulin ([1]) assured me (half-jokingly) that in Harvest and Sowing I was doing the "ethnology of the mathematical milieu" (or perhaps the sociology, I don't know). One is flattered, of course, when he learns that (without even knowing it) he is doing something scholarly! It's a fact that during the "enquiry" part of the reflection (and against my will...), I have passed through, in the pages I was writing, a good part of the mathematical establishment, not to mention a number of colleagues and friends of more modest status. And in the last few months, since I dispatched the provisional copies of Harvest and Sowing last October, it has "happened again". Clearly, my testimony came as a fly in the ointment. There were echoes on all sides surely (save that of boredom...). But almost every time, it was not at all what I had expected. And there was also a lot of silence, which says a lot. Obviously, I had (and still have) a lot to learn, all sorts of things, about what goes on in the noggin of the various people, among my ex-students and other more or less well situated colleagues - sorry, I mean, about the 'sociology of the mathematical milieu'! To all those who have already come to make their contribution to the great sociological work of my old age, I wish to express here my grateful feelings.

Of course, I was particularly touched by the echoes in hearty tones. There were also some rare colleagues who told me about an emotion, or a feeling (unexpressed until then) of crisis, or of degradation within this mathematical milieu which they feel to be part of.

Outside this milieu, among the very first to give a hearty, even emotional, welcome to my testimony, I would like to mention Sylvie and Catherine Chevalley([2]), Robert Jaulin, Stéphane Deligeorge and Christian Bourgois. If Harvest and Sowing is going to be distributed more widely than the initial provisional copies (intended for a very restricted circle), it is above all thanks to them. Thanks, above all, to their communicative conviction: that what I have tried to grasp and say had to be said. And that it could be heard in a wider circle than that of my colleagues (often sullenly, even aggressively, and not at all willing to question themselves...). Thus Christian Bourgois did not hesitate to run the risk of publishing the unpublishable, and Stéphane Deligeorge did me the honour of including my indigestible testimony in the "Epistémè" collection, alongside (for the moment) Newton, Cuvier and Arago. (I could not have wished for better company!) To each and every one of them, for

[1] Robert Jaulin is an old friend. I understand that he is in a somewhat similar position (as a "white wolf") vis-à-vis the establishment in the ethnological milieu as I am vis-à-vis the mathematical "beautiful world".

[2] Sylvie and Catherine Chevalley are the widow and daughter of Claude Chevalley, the colleague and friend to whom the central part of Harvest and Sowing (ReS III, "The Key to Yin and Yang") is dedicated. In several places in the reflection, I speak of him, and of the role he played in my journey.

their repeated expressions of sympathy and confidence, coming at a particularly "sensitive" moment, I am happy to express here all my gratitude.

And here we are at the start of a Promenade through a work, as an introduction to a voyage through a life. A long journey, yes, of more than a thousand pages, and each one well packed. It took me a lifetime to make this journey, without having exhausted it, and more than a year to rediscover it, page after page. Words have sometimes been reluctant to come, to express all the juice of an experience still evading a hesitant understanding - as ripe and thick grapes piled up in the press seem, at times, to want to evade the force that embraces them... But even in those moments when words seem to jostle and flow, it is not to the happy-go-lucky that they jostle and flow. Each of them has been weighed in passing, or perhaps afterwards, to be carefully adjusted if it has been found too light, or too heavy. So this reflection-testimony-voyage is not meant to be read quickly, in a day or in a month, by a reader who is in a hurry to get to the final word. There are <u>no</u> "final words", no "conclusions" in Harvest and Sowing, any more than there are in my life, or in yours. There is a wine, aged for a lifetime in the barrels of my being. The last glass you drink will not be better than the first or the hundredth. They are all "the same", and they are all different. And if the first glass is spoiled, the whole cask is spoiled; then you might as well drink good water (if there is any), rather than bad wine.

But good wine is not to be drunk in a hurry, nor on the run.

Promenade through an œuvre - or The Child and his Mother

January 1986

1. The Magic of Things

When I was a kid, I liked to go to school. We had the same teacher to teach us to read and write, arithmetic, singing (he played a little violin to accompany us), or prehistoric man and the discovery of fire. I don't remember ever being bored at school at that time. There were the magic of numbers, and the magic of words, signs and sounds. Also the <u>rhymes</u>, in the songs or little poems. There seemed to be a mystery in the rhyme beyond the words. It was like that, until one day someone explained to me that there was a very simple "trick"; that rhyme is simply when you make two consecutive speakings end with the same syllable, which then, as if by magic, become verses. It was a revelation! At home, where I could find people to talk to, for weeks and months on end I would have fun making verses. At one point, I could only speak in rhymes. Fortunately, I got over that. But even today I still occasionally write poems - but I don't really go looking for the rhyme anymore, if it doesn't come by itself.

At another time an older friend, who was already in the lycée, taught me negative numbers. This was another amusing game, but it was soon exhausted. And there was the crossword puzzle - I spent days and weeks making them, more and more interlocking. This game combined the magic of form, with the magic of signs and words. But that passion left me, apparently without leaving any trace.

In high school, first in Germany, then in France, I was a good student, but not the 'brilliant student'. I invested myself wholeheartedly in what interested me most, and tended to neglect what interested me less, without worrying too much about the appreciation of the "prof" concerned. The first year of lycée in France, in 1940, I was interned with my mother in the concentration camp at Rieucros near Mende. It was wartime, and we were foreigners - "undesirables", as they called. But the camp administration kept an eye on the kids in the camp, however undesirable they were. You could come and go as you pleased. I was the oldest, and the only one to go to the lycée, four or five kilometres away, whether it was snowing or windy, with makeshift shoes that always got wet.

I still remember the first "maths composition", where the teacher gave me a bad mark, for the proof of one of the "three cases of equality of triangles". My proof was not the one in the book, which he followed religiously. However, I knew perfectly well that my proof was neither more nor less convincing than the one in the book, the spirit of which I was following, with the endless traditional "we slide such and such a figure over such and such a figure". Obviously, this man who was teaching me did not feel capable of judging by his own lights*[lumières]* (here, the validity of a reasoning). He had to refer to an authority, in this case a book. It must have struck me, these dispositions, for me to have remembered this little incident. Afterwards, and even today, I have had ample opportunity to see that such dispositions are by no means the exception, but an almost universal rule. There is a lot to be said for this - a subject I touch upon more than once in one form or another in Harvest and Sowing. But even today, whether I like it or not, I feel disconcerted every time I am confronted with it again...

In the last years of the war, while my mother remained interned in the camp, I was in a "Secours Suisse" children's home for refugee children in Chambon sur Lignon. Most of us were Jewish, and when we were warned (by the local police) that there would be Gestapo raids, we would go and hide in the woods for a night or two, in small groups of two or three, without really realising that our lives were at stake. The region was full of Jews hiding in the Cévenol region, and many survived thanks to the solidarity of the local community.

What struck me most at the "Collège Cévenol" (where I was a student) was how little interest my fellow students had in what they were learning. As for me, I devoured the textbooks at the beginning of the school year, thinking that this time, we would finally learn something <u>really</u> interesting; and the rest of the year I used my time as best I could, while the planned program was inexorably churned out, term after term. We had some really nice profs though. The natural history prof, Mr. Friedel, was of a remarkable human and intellectual quality. But, unable to "crack down*[sévir]*", he was heckled to death, to the point that towards the end of the year, it became impossible to follow, his impotent voice covered by the general hullabaloo. That's why, perhaps, I didn't become a biologist!

I spent a lot of my time, even during lessons (shh...), doing math problems. Soon the ones in the book were not enough for me. Maybe because they tended to resemble each other a little too much; but mostly, I think, because they fell out of the blue a little too much, just like that, without saying where they came from or where they went. These were the problems of the book, not <u>my</u> problems. Yet there was no shortage of really natural questions. For example, when the lengths a, b, c of the three sides of a triangle are known, the triangle is known (apart from its position), so there must be an explicit "formula" to express, for example, the area of the triangle as a function of a, b, c. The same goes for a tetrahedron where the length of the six edges is known - what is the volume? I think I had to struggle with this one, but I must have managed to do it in the end. Anyway, when something "stuck" to me, I never counted the hours or the days I spent on it, even if it meant forgetting everything else! (And it's still like that now...)

What I was least satisfied with in our math books was the absence of any serious definition of the notion of length (of a curve), of area (of a surface), of volume (of a solid). I promised myself to fill this gap, as soon as I had the time. I spent most of my energy there between 1945 and 1948, when I was a student at the University of Montpellier. The courses at the University were not satisfying for me. Without ever having said it explicitly, I must have had the impression that the teachers were just repeating their books, just like my first math teacher at the lycée in Mende. So I only went to college from time to time, to keep up with the endless "programme". The books were enough for the programme, but it was also clear that they did not answer the questions I had. In fact, they didn't even <u>see</u> them, any more than my lycée books saw them. As long as they gave recipes for calculating lengths, areas and volumes, using simple, double and triple integrals (dimensions greater than three were cautiously avoided...), the question of giving an intrinsic definition did not seem to arise, either for my teachers or for the authors of the textbooks.

From the limited experience I had at that time, it might well seem that I was the only person in the world gifted with a curiosity for mathematical questions. Such was my unspoken conviction, during those years spent in complete intellectual solitude, which did not weigh on me([1]). To tell the truth, I think that I never thought, during this time, to consider the question whether or not I was the only person in the world likely to be interested in what I was doing. My energy was sufficiently absorbed in the challenge I had set myself: to develop a theory that would satisfy me fully.

There was no doubt in my mind that I could not fail to get there, to find the ultimate answer, if only I took the trouble to scrutinise them, putting down in black and white what they were telling me, progressively step by step. The intuition of the volume, let's say, was unquestionable. It could only be the reflection of a reality, elusive for the moment, but perfectly reliable. It is this reality that it was a question of seizing, quite simply - perhaps a little like this magic reality of "the rhyme", which had been seized and "understood" before.

When I started, at the age of seventeen and fresh out of lycée, I thought it would be a matter of just a few weeks. I stayed on it for three years. I even managed, by dint of it, to flunk an exam, at the end of my second year of college - the one of spherical trigonometry (in the "advanced astronomy" option, sic), because of a silly mistake in numerical calculation. (I was never very good in calculations, I must say, since I was out of lycée...) That's why I had to stay a third year in Montpellier to finish my degree, instead of going to Paris right away - the only place, I was assured, where I would have the opportunity to meet people who knew what was considered important in math. My informant, Mr. Soula, also assured me that the last problems existed in math had been solved, twenty or thirty years ago, by a man named Lebesgue. He had developed precisely (really a strange coincidence!) a theory of measure and integration, which put an end point to mathematics.

Mr. Soula, my "calculus diff" prof, was a kind man and well disposed towards me. I don't believe that he convinced me for all that. There must have been already in me the prescience that mathematics is a thing unlimited in extent and depth. Does the sea have an "end point"? Still, at no time was I touched by the thought of going to find the book of this Lebesgue of which Mr. Soula had spoken to me, which he must have never held in his hands either. In my mind, there was nothing in common between what a book could contain, and the work I was doing, in my own way, to satisfy my curiosity about things that had intrigued me.

2. The importance of being alone

When I finally got in touch with the mathematical world in Paris, one or two years later, I ended up learning, among many other things, that the work I had done in my corner with the means at hand, was (more or less) what was well known to "the whole world", under the name of "theory of Lebesgue measure and integration".

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[1] Between 1945 and 1948, I lived with my mother in a small hamlet about ten kilometers from Montpellier, Mairargues (by Vendargues), hidden in the midst of the vineyards. (My father had disappeared in Auschwitz, in 1942.) We lived meagerly on my meager student scholarship. To make ends meet, I picked grapes every year, and after the pick, I made grape wine, which I managed to sell as best I could (in contravention, it seemed, of the legislation in force...) In addition, there was a garden which, without ever having to work on it, provided us with an abundance of figs, spinach, and even (towards the end) tomatoes, planted by a complaisant neighbour in the midst of a sea of splendid poppies. It was a good life - but sometimes it became really inconvenient, when it was necessary to replace a pair of glasses or a pair of shoes that were worn out to the bone. Fortunately, my mother, who was weak and sick from her long stay in the camps, was entitled to free medical assistance. We would never have been able to afford a doctor...

In the eyes of the two or three seniors to whom I talked about this work (even showed a manuscript), it was a bit like if I had just wasted my time, re-doing what was "already known". Yet I don't remember being disappointed, by the way. At that time, the idea of getting "credit", or even approval or simply interest from others, for the work I was doing, must have been still foreign to my mind. Not to mention that my energy was quite busy getting acquainted with a completely different milieu, and above all, learning what was considered in Paris as the mathematician's B.A.BA([1]).

Yet, looking back on those three years, I realise that they were by no means wasted. Without even knowing it, I learned then in solitude what is the essence of being a mathematician - what no master can really teach. Without ever telling myself, without meeting anyone with whom I could share my thirst for understanding, I knew nevertheless, "by my gut" I would say, that I was a mathematician: someone who "does*[fait]*" maths, in the full sense of the word - like one "makes*[fait]*" love. Mathematics had become for me a mistress always welcoming my desire. These years of solitude laid the foundation of a confidence that was never shaken - neither by the discovery (arriving in Paris at the age of twenty) of the full extent of my ignorance and the immensity of what I had to learn; nor (more than twenty years later) by the turbulent episodes of my departure without return from the mathematical world; nor, in these last years, by the often quite crazy episodes of a certain "Burial" (anticipated and no mistake) of my person and my work, orchestrated by my closest companions of yesteryear...

To put it differently: I have learned, in those crucial years, to <u>be alone</u>([2]). By this I mean: to approach by my own lights the things I want to know, rather than to rely on the ideas and the consensus, expressed or tacit, which come to me from a large or small group which I feel to be a member of, or which for any other reason would be vested upon me with authority. Silent consensuses had told me, in lycée as in university, that there was no need to question the very notion of "volume", presented as "well known", "obvious", "without problem". I had passed over it, as a matter of course - just as Lebesgue, a few decades earlier, had passed over it. It is in this act of "<u>passing over</u>", of being oneself, in short, of not simply repeating the consensus which is the law, of not remaining locked up inside the imperative circle that people set for us - it is above all in this solitary act that “<u>the creation</u>" is found. All the rest comes along with it*[par surcroit]*.

Afterwards, I had the opportunity, in this world of mathematicians that welcomed me, to meet many people, both seniors and young people more or less my age, who were obviously much more brilliant, much more "gifted" than me. I admired them for the ease with which they learned, as if by playing, new notions, and juggled with them as if they had known them since their cradle - whereas I felt heavy and clumsy, painfully making my way, like a mole, through a shapeless mountain of things which were important (I was assured) for

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[1] I give a short account of this rough transition time in the first part of Harvest and Sowing (ReS I), in the section "The welcomed stranger" (no. 9).

[2] This formulation is somewhat improper. I never had to "learn to be alone", for the simple reason that I never <u>unlearned</u>, during my childhood, this innate capacity which was in me at my birth, as it is in everyone. But these three years of solitary work, where I was able to give my full measure to myself, according to the criteria of spontaneous demand which were my own, confirmed and rested in me, in my relation at this very time to mathematical work, a foundation of confidence and quiet assurance, which owed nothing to the consensus and fashions which make the law. I will have the occasion to allude to it again in the note "Roots and solitude" (ReS IV, no. 171.3, notably p. 1080).

me to learn, and of which I felt unable to grasp the ins and outs. In fact, I had nothing of the brilliant student, passing with ease prestigious competitions, assimilating in no time prohibitive programmes.

Most of my brighter comrades have become competent and renowned mathematicians. However, with the hindsight of thirty or thirty-five years, I see that they have not left a really deep imprint on the mathematics of our time. They have done things, sometimes beautiful things, in a context that was already ready-made, and which they never think of disturbing. They remained prisoners without knowing of these invisible and imperious circles, which delimit a Universe in a milieu and at a given time. To cross them, they would have had to find in themselves this capacity which was theirs at their birth, just as it was mine: the capacity to be alone.

The small child, however, has no difficulty in being alone. He is solitary by nature, even if he doesn't mind the occasional company and knows how to ask for his mother's breast*[totosse]* when it's time to drink. And he knows, without having to say this to himself, that the breast is for him, and that he knows how to drink. But often, we have lost touch with this child in us. And all the time we miss the best things, without deigning to see them...

If in Harvest and Sowing I am addressing to someone else besides myself, it is not to a "public". I am addressing to you, who are reading me, as a person, a sole person. It is to the one in you who knows how to be alone, to the child, that I would like to speak, and to nobody else. The child is often far away, I know this well. He has seen all kinds of things and for a long time. He hid himself God knows where, and it's not easy, often, to get to him. You would swear he's been dead forever, that he never existed - and yet, I'm sure he's out there somewhere, alive and well.

And I also know what is the sign that I am heard. It is when, beyond all the differences of culture and destiny, what I say about myself and my life finds in you an echo and a resonance; when you also find in it your own life, your own experience of yourself, in a light perhaps to which you had not paid attention until then. It is not a question of "identification", with something or someone far from you. But perhaps, in a way, you are rediscovering your own life, what is closest to you, through the rediscovery that I am making of mine, through the pages in Harvest and Sowing and even these pages that I am writing today.

3. The interior adventure - or myth and testimony

Above all, Harvest and Sowing is a reflection on myself and my life. By the same token, it is also a testimony, in two ways. It is a testimony about my past, on which the main weight of the reflection is placed. But at the same time it is also a testimony to the most immediate present - to the very moment when I am writing, and when the pages of Harvest and Sowing are born in the course of hours, nights and days. These pages are the faithful witnesses of a long meditation on my life, as it has really happened (and is still happening at this very moment...).

These pages have no literary pretensions. They constitute a document on myself. I have only allowed myself to touch them (for occasional stylistic alterations, for example) within very narrow limits([1]). If it has any pretension, it is only to be true. And there is a lot of it.

This document, by the way, is not an "autobiography". You will not learn my date of birth (which would be of little interest except for an astrological chart), nor the names of my mother and father or what they did for a living, nor the names of my wife and other women who were important in my life, or the names of the children born of these loves, and what they did with their lives. It is not that these things were not important in my life, or do not remain important now. But as this self-reflection went on and on, at no time have I felt prompted to engage in any description of the things I've come across here and there, much less to dutifully line up names and numbers. At no time did it seem to me that this would add anything to the point I was pursuing at the moment. (While in the few pages above, I have been led, as if against myself, to include perhaps more material details about my life than in the thousand pages below...)

And if you ask me what is this "point" that I am pursuing for a thousand pages, I will answer: it is to tell the story, and thus the discovery, of the inner adventure that my life has been and still is. This narrative-testimony of an adventure continues at the same time on the two levels I have just mentioned. There is the exploration of an adventure in the past, of its roots and its origin in my childhood. And there is the continuation and renewal of this "same" adventure, as I write Harvest and Sowing, in spontaneous response to a violent interpellation from the outside world([2]).

The external facts come to feed the reflection, only insofar as they arouse and provoke a rebound of the interior adventure, or contribute to enlighten it. And the burial and the plundering of my mathematical work, which will be discussed at length, was such a provocation. It provoked in me a mass raising of powerful egotistical reactions, and at the same time revealed to me the deep and ignored links which continue to connect me to the work coming out of me.

It is true that the fact that I am one of those "strong in maths" is not necessarily a reason (and even less a good reason) for you to be interested in my particular "adventure" - nor is the fact that I have had trouble with my colleagues, after having changed milieu and lifestyle. There is no shortage of colleagues or even friends, who find it ridiculous to display in public (as they say) their “states of mind*[états d’âme]*". What counts, are the "results". The “soul*[âme]*", that is to say the thing in us which lives through the "production" of these "results", and also its repercussions of all kinds (in the life of the "producer", as much as in that of his fellow men), is the object of disdain, even of an openly displayed derision. This attitude is an expression of "modesty". I see in it the sign of an escape, and a strange derangement, promoted by the very air we breathe. It is certain that I do not write for the one struck by this sort of latent contempt of himself, which makes him disdain the best I have to offer. A contempt for what really makes his own life, and for what makes mine: the superficial and deep, coarse or subtle movements which animate the psyche, this very "soul" which lives through the experience and reacts to it, which freezes or blossoms, which retreats or learns...

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[1] Thus, the possible rectification of errors (of materials, perspectives, etc.) are not by altering the first draft, but are done in footnotes, or during a later "return" to the situation under examination.

[2] For details about this "violent interpellation", see "Letter", especially sections 3 to 8.

The story of an inner adventure can only be told by the one who lives it, and by no other. But even if the story were intended only for oneself, it is rare that it does not glide into the rut of the construction of a myth, of which the narrator would be the hero. Such a myth is born, not from the creative imagination of a people and a culture, but from the vanity of one who does not dare to assume a humble reality, and who likes to substitute a construction, a work of his mind. But a true story (if there is one), of an adventure as it was really lived, is a precious thing. And this, not by a prestige that (rightly or wrongly) would surround the narrator, but by the mere fact of its existence, with its quality of truth. Such a testimony is precious, whether it comes from a man of reputation or even fame, or from a small employee with no future and laden with a family, or from a criminal of common law.

If such a story has a virtue for others, it is above all to confront them with themselves, through this unvarnished testimony of another's experience. Or also (to say it differently) to perhaps erase in him (even if only during the timespan of his reading) this contempt in which he holds for his own adventure, and this "soul" which is the passenger as well as the captain...

4. Picture of manners

While talking about my past as a mathematician, and subsequently discovering (as if unwillingly) the twists and turns of the gigantic Burial of my work, I was led, without having sought it, to paint a picture of a certain milieu and a certain era - of an era marked by the breakdown of certain values which gave meaning to the work of men. This is the aspect of the "picture of manners*[mœurs]*", painted around "true events" which is undoubtedly unique in the annals of "Science". What I said before, should make it quite clear, I think, that you will not find in Harvest and Sowing a "dossier" concerning a certain unusual "affair", just to quickly bring you up to date. Any friend who, however, searches this dossier, will have passed through with his eyes closed, not seeing anything which makes up almost all of the substance and flesh of Harvest and Sowing.

As I will explain in a much more detailed way in the Letter, the "investigation" (or the "picture of manners ") is pursued especially during parts II and IV, "The Burial (1) - or the robe of the Chinese Emperor" and "The Burial (3) - or the Four Operations". As the pages go by, I obstinately bring to light, one after the other, a multitude of juicy facts (to say the least), which I try as best I can to "file up*[caser]*" as I go along. Little by little, these facts come together in an overall picture that gradually emerges from the mists, in brighter and brighter colours, with sharper and sharper contours. In these day-to-day notes, the "raw facts" that have just appeared are inextricably mixed with personal reminiscences, and with comments and reflections of a psychological, philosophical, and even (occasionally) mathematical nature. That's just the way it is and I can't help it!

From the work that I have done, which has kept me on the edge of my seat for more than a year, putting together a dossier, in the style of "investigation's conclusions", should require some additional work of the scale of a few hours or a few days, depending on the curiosity and the exigency of the readers interested. I did try at one point to put it together, this famous dossier. It was when I started to write a note that was to be called "The

Four Operations"([1]). Yet then, no, nothing could be done. I couldn't do it! This is not my style of expression, definitely, and less than ever in my old age. And I think now, with Harvest and Sowing, that I have done enough for the benefit of the "mathematical community", to leave without remorse to others (if there are any among my colleagues who would feel concerned) the task of putting together the necessary "dossier".

5. The inheritors and the builder

It is time for me to say a few words here about my mathematical work, which has taken and still takes (to my own surprise) an important place in my life. More than once in Harvest and Sowing I will come back to this work - sometimes in a way clearly intelligible to everyone, and at other times in somewhat technical terms([2]). These latter passages will largely go "over the head" not only of the "layman", but even of the fellow mathematician who is more or less "in the know" about the maths discussed. You can of course skip the passages that seem to you to be a bit too "sophisticated*[calée]*". You can also browse through them, and perhaps catch a glimpse of the "mysterious beauty" (as a non-mathematician friend wrote to me) of the world of mathematical things, appearing like so many "strange inaccessible islands" in the vast moving waters of the reflection...

Most mathematicians, as I said earlier, are inclined to confine themselves to a conceptual framework, to a "<u>Universe</u>" fixed once and for all - the one, essentially, that they found "ready-made" at the time they did their studies. They are like the inheritors of a big and beautiful house all set up, with its living rooms and its kitchens and its workshops, and its cookware and tools of all kinds, with which there is, well, enough to cook and tinker. How this house has been built over the generations, and how and why such and such tools have been designed and made (and not others...), why the rooms are arranged and furnished in such and such a way here, and in such and such a way there - these are all questions that these inheritors would never think to ask themselves. This is the "Universe", the "given" in which one must live, and that' s all! Something that seems large (and one is, most of the time, far from having gone around all its parts), but <u>familiar</u> at the same time, and above all: <u>immutable</u>. When they are busy, it is to maintain and embellish a heritage: to repair a wobbly piece of furniture, to plaster a facade, to sharpen a tool, or even sometimes, for the most enterprising, to make a new piece of furniture from scratch in the workshop. And it happens, when they put all their energy into it, that the furniture is beautiful and that the whole house looks more beautiful.

Even more rarely, one of them will think of making some modification to one of the tools of the stock, or even, under repeated and insistent pressure of needs, of imagining and making a new one. Being able to do this only if he doesn’t confuse himself in making apologies, for what he feels as a kind of infringement of the piety of the family tradition, which he has the impression of upsetting by an unusual innovation.

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[1] The planned note ended up bursting into part IV (of the same name "The four operations") of Harvest and Sowing, comprising in the 70 notes extending on well over four hundred pages.

[2] There are also here and there, in addition to mathematical insights into my past work, passages containing new mathematical developments. The longest is "The five pictures (crystals and D-Modules)" in ReS IV, note no. 171 (ix).

In most rooms of the house, the windows and shutters are carefully closed - no doubt for fear of a wind coming from elsewhere.And when the beautiful new furniture, one here and one there, not to mention the offspring, begins to clutter up the narrowed rooms and invade the hallways, none of these inheritors will want to realise that their familiar and cozy world is getting a little tight around the edges. Rather than resigning themselves to such an observation, some will prefer to sneak and wedge themselves as best they can, between a Louis XV sideboard and a rattan rocking chair, between a snotty baby and an Egyptian sarcophagus, and others, in desperation, will climb as best they can over a crumbling heap of chairs and benches...

The little picture I have just painted is not special to the world of mathematicians. It illustrates inveterate and immemorial conditioning, which can be found in all milieus and in all spheres of human activity, and also (as far as I know) in all societies and at all times. I have already alluded to this, and I do not claim to be free of it myself. As my testimony will show, the opposite is true. It only happens that at the relatively limited level of an intellectual creative activity, I have been rather unaffected([1]) by this conditioning, which could be called "cultural blindness" - the inability to see (and to move) outside the "Universe" fixed by the surrounding culture.

As for me, I feel that I belong to the line of mathematicians whose spontaneous vocation and joy is to constantly build new houses([2]). Along the way, they can't help but invent and shape all the tools, utensils, furniture and instruments required to build the house from the foundation to the ridge, to provide in abundance for the future kitchens and workshops, and to set up the house to live in and feel comfortable in. However, once everything has been laid down to the last gutter and the last stool, it is rare that the worker lingers long in these places, where each stone and each rafter bears the trace of the hand that worked on it and laid it. His place is not in the quietude of ready-made universes, however welcoming and harmonious they may be - whether they have been arranged by his own hands, or those of his predecessors. Other tasks already call him to new building sites, under the imperious impulse of needs which he is perhaps the only one to feel clearly, or (more often still) by anticipating needs which he is the only one to sense. His place is in the open air. He is a friend of the wind and is not afraid of being alone in his task, for months and years and, if necessary, for a whole life, if a welcomed relief does not come to the rescue. He has only two hands like everyone else, of course - but two hands that always sense what they have to do, that do not shy away from the biggest and most delicate tasks, and that never tire of getting to know and reacquaint themselves with the countless things that constantly call upon them to know them. Two hands is not much, perhaps, because the World is infinite. They will never exhaust it! And yet, two hands, is a lot..

I, who am not strong in history, if I had to give names of mathematicians in this lineage, Galois and Riemann (in the last century) and Hilbert (at the beginning of the present century) come to me spontaneously. If I look for a representative among the seniors who welcomed me at my beginnings in the mathematical world([3]), it is the

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[1] I think the main reason for this is a certain favourable climate that surrounded my childhood until the age of five. See on this subject the note "Innocence" (ReS III, no. 107).

[2] This archetypal image of the "house" to be built, surfaces and is formulated for the first time in the note "Yin the Servant, and the new masters" (ReS III, no. 135).

[3] I discuss these beginnings in the section "The welcomed stranger" (ReS I, no. 9).

name of Jean Leray that comes to me before any other, even though my contacts with him have remained very episodic([1]).

I have just sketched two portraits: that of the "homebound" mathematician who is content to maintain and embellish a heritage, and that of the pioneer-builder([2]), who cannot stop himself from constantly crossing those "invisible and imperious circles" which delimit a Universe([3]). They can also be called, by slightly offhand*[emporte-pièce]* but suggestive names, the "conservatives" and the "innovators". Both have their reason for being and their role to play, in the same collective adventure continuing over generations, centuries and millennia. In a period when a science or an art is flourishing, there is no opposition or antagonism([4]) between these two temperaments. They are different and they complement each other as the dough and the leaven complement each other.

Between these two extreme types (but by no means opposed in nature), there is of course a whole range of intermediate temperaments. A "homebound" who would never dream of leaving a familiar home, and even less of going to the trouble of building another God knows where, will not hesitate, however, when things start to get cramped, to put his hand to the trowel to fit out a cellar or an attic, to raise a floor, or even, if need be, to add to the walls some new outbuilding of modest proportions ([5]). Without being a builder at heart, he often looks with sympathetic eyes, or at least without secret concern or reprobation, at another person who had shared the same dwelling with him, and who is now struggling to put together beams and stones in some impossible outback, with the air of someone who already sees a palace there...

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[1] This does not prevent me from being (following H. Cartan and J. P. Serre) one of the main users and promoters of one of the great innovative notions introduced by Leray, that of sheaves, which has been one of the essential tools throughout my work as a geometer. It is also the one that provided me with the key to the enlargement of the notion of (topological) space into that of topos, which will be discussed later.

Leray differs from the portrait I have drawn of the "builder", it seems to me, in that he does not seem to be inclined to "build houses from the foundations to the ridge". Rather, he could not help but lay vast foundations, in places that no one else would have thought of, while leaving it to others to finish them and build on them, and, once the house was built, to settle down in it (if only for a while)...

[2] I have just, surreptitiously and 'by the by', added two terms with male resonance (that of 'builder' and that of 'pioneer'), which nevertheless express very different aspects of the impulse to discover, as well as a nature more delicate than these names could evoke. This is what will become apparent in the next part of this promenade-reflection, in the stage "Discovering the Mother - or the two sides" (no. 17).

[3] At the same time, moreover, and without having intended it, he assigns to this old Universe (if not for himself, at least for his fellow men who are less mobile than he is) new limits, in new circles that are certainly wider, but just as invisible and just as imperious as those they have replaced.

[4] This was particularly the case in the mathematical world, during the period (1948-1969) that I witnessed first hand, when I was part of that world myself. After I left in 1970, there seems to have been a kind of wide-ranging reaction, a kind of "consensus of disdain" for "ideas" in general, and more particularly, for the great innovative ideas I had introduced.

[5] Most of my "seniors" (referred to, for example, in "A welcomed debt", Introduction, 10) correspond to this intermediate temperament. I am thinking in particular of Henri Cartan, Claude Chevalley, André Weil, Jean-Pierre Serre, Laurent Schwartz. With the possible exception of Weil, they all gave a 'sympathetic eye', without 'secret concern or disapproval', to the solitary adventures they saw me embarking on.

6. Points of view and vision

But let me I come back to myself and my work.

If I have excelled in the art of being a mathematician, it is not so much because of the skill and perseverance in solving problems bequeathed to me by my predecessors, as because of that natural propensity in me which leads me to see questions, which are clearly crucial, yet no one had seen, or to bring out the "good notions" which were missing (usually not realised by anybody, before the new notion appeared), as well as the "good statements" which no one had thought of. Very often, notions and statements fit together so perfectly that there can be no doubt in my mind that they are correct (with a few alterations, at most) - and often then, when it is only a "work in progress" intended for publication, I dispense with going further, and taking the time to work out a proof which very often, once the statement and its context are clearly seen, can hardly be more than a matter of "craft*[métier]*", or even of routine. There are countless things that demand attention, and it is impossible to follow the call of each one to the end! This does not prevent the fact that the propositions and theorems duly proved, in my written and published work, number in the thousands; and I think I can say that, with very few exceptions, they have all become part of the common heritage of things commonly accepted as "known" and commonly used almost everywhere in mathematics.

But even more than towards the discovery of new questions, notions and statements, it is towards the discovery of fertile points of view, constantly leading me to introduce, and to develop more or less, entirely new themes, that my particular genius leads me. This, it seems to me, is the most essential contribution I have made to the mathematics of my time. To tell the truth, these innumerable questions, notions, and statements I have just mentioned only take on meaning for me in the light of such a "point of view" - or to put it better, they emerge spontaneously from it, with the force of evidence; in the same way that a light (even a diffuse one) which emerges in the dark night, seems to give rise to those more or less fuzzy or sharp outlines which it suddenly reveals to us. Without this light which unites them in a common bundle*[faisceau]*, the ten or hundred or thousand questions, notions, statements would appear as a heterogeneous and amorphous heap*[monceau]* of "mental gadgets", isolated from one another - and not as parts of a Whole which, although perhaps remaining invisible, still hiding in the folds of the night, is nevertheless clearly foreseen.

The fertile point of view is the one that reveals to us, like so many living parts of the same Whole that encompasses them and gives them meaning, those burning questions that no one felt, and (as if in response perhaps to these questions) those notions that are so natural that no one had thought to draw out, as well as those statements that seem to flow from the source, and which certainly no one risked asking, as long as the questions that gave rise to them, and the notions that make it possible to formulate them, had not yet appeared. Even more than the so-called "key theorems" in mathematics, it is the fruitful points of view that are, in our art([1]), the most powerful tools of discovery - or rather, they are not tools, but the very eyes of the researcher which, passionately, wants to know the nature of mathematical things.

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[1] This is certainly not the case in "our art" alone, but (it seems to me) in all work of discovery, at least when it is on the level of intellectual knowledge.

Thus, the fruitful point of view is none other than that "eye" which both makes us discover, and makes us recognise the unity in the multiplicity of what is discovered. And this unity is truly the very life and breath that links and animates these multiple things.

But as its very name suggests, a "point of view" in itself remains fragmentary. It reveals to us one aspect of a landscape or a panorama, among a multiplicity of other equally valid, equally "real" aspects. It is insofar as the complementary points of view of the same reality are combined, in which our "eyes" are multiplied, that the gaze penetrates further into the knowing of things. The richer and more complex the reality we wish to know, the more important it is to have several "eyes"([1]) to apprehend it in all its breadth, all its finesse.

And it happens, sometimes, that a bundle of converging points of view on the same vast landscape, by virtue of that in us which is able to grasp the One through the many, gives shape to a new thing; to a thing which exceeds each of the partial perspectives, in the same way that a living being exceeds each of its limbs and organs. This new thing, one may call a vision. The vision unites the already known points of view that embody it, and it reveals to us others that were previously ignored, just as the fruitful point of view makes us discover and apprehend as part of the same Whole, a multiplicity of new questions, notions and statements.

To put it another way: vision is to the points of view from which it appears to come and which it unites, as clear and warm daylight is to the different components of the solar spectrum. A vast and profound vision is like an inexhaustible source, made to inspire and enlighten the work not only of the one in whom it was born one day and who made himself its servant, but of generations, fascinated perhaps (as he was himself) by those distant limits which it gives us a glimpse of...

7. The "great idea" - or the trees and the forest

The so-called 'productive' period of my mathematical activity, that is to say, the one attested by publications in due form, extends from 1950 to 1969, about twenty years. And for twenty-five years, between 1945 (when I was seventeen) and 1969 (when I was about forty-two), I invested practically all my energy in mathematical research. An inordinate investment, to be sure. I paid for it by a long spiritual stagnation, by a progressive 'thickening', which I will have more than one occasions to discuss in the pages of Harvest and Sowing. However, within the limited field of a purely intellectual activity, and through the blossoming and maturation of a vision restricted to the world of mathematical things alone, these were years of intense creativity.

During this long period of my life, almost all my time and energy was devoted to what is called "work on pieces*[travail sur pièces]*": to the painstaking work of shaping, assembling and grinding, required for the construction of all the pieces of the houses that an inner voice (or demon...) was telling me to build, following a masterpiece that it was whispering to me as the work progressed. Taken up with the tasks of the "craft": those of stonecutter, mason, carpenter, and even the plumber, joiner and cabinetmaker - I rarely took the time to note

[1] Every point of view leads to the development of a language that expresses it and which is specific to it. Having several 'eyes' or several 'points of view' to apprehend a situation also means (in mathematics at least) having several different languages to grasp it.

down in black and white, even if only in broad strokes, the master plan invisible to all (and it became apparent later...) except me, which over the course of days, months and years guided my hand with the surety of a sleepwalker([1]). It must be said that the work on pieces, in which I liked to put a loving care, was not at all to my displeasure. Moreover, the mathematical mode of expression that was taught and practised by my elders gave pre-eminence (to say the least) to the technical aspect of the work, and hardly encouraged 'digressions' that would have lingered on the 'motivations'; or even those which would have pretended to bring out of the mists some image or vision which might have been inspiring, but which, since it had not yet been embodied in tangible constructions in wood, stone or pure and hard cement, was more akin to shreds of a dream than to the work of a diligent and conscientious craftsman.

On the quantitative level, my work during these years of intense productivity has materialised above all in some twelve thousand pages of publications, in the form of articles, monographs or seminars([2]), and in hundreds, if not thousands, of new notions, which have entered into the common heritage, with the very names that I had

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[1] The image of the "sleepwalker" was inspired by the title of Koestler's remarkable book “The sleepwalkers" (Calman*[Calmann -Trans.]* Lévy), presenting an "Essay on the history of conceptions of the Universe", from the origins of scientific thoughts up to Newton. One of the aspects of this story that struck Koestier and which he brought to light is, how often, the path from a certain point in our knowledge of the world to some other point which (logically and with hindsight) seems very close, passes through the sometimes most preposterous detours, which seem to defy sound reason; and yet, through these thousand detours which seem to lead them astray forever, and with a "sleepwalking certainty", men who set out in search of the "keys" to the Universe stumble, as if unwillingly and without even realising it, upon <u>other</u> "keys" which they were far from foreseeing, and which nevertheless turn out to be "the right ones".

From what I have observed around me, at the level of mathematical discovery, these outrageous detours in the path of discovery are made by some large-scale researchers, but by no means by all. This may be due to the fact that for the past two or three centuries, research in the natural sciences, and especially in mathematics, has been freed from the imperative religious or metaphysical presuppositions of a given culture and time, which have been particularly powerful obstacles to the unfolding (for better or for worse) of a 'scientific' understanding of the Universe. It is true, however, that some of the most fundamental and obvious ideas and notions in mathematics (such as direct isometry, group, number zero, arithmetic, coordinates of a point in space, notion of a set, or the notion of a topological 'form', not to mention negative numbers and complex numbers) took thousands of years to make their appearance. These are all eloquent signs of this inveterate "block", deeply implanted in the psyche, against the conception of totally new ideas, even in cases where these are childishly simple and seem to impose themselves with the force of evidence, for generations, or even millennia...

Coming back to my own work, I have the impression that in it the "blunders*[foirages]*" (more numerous perhaps than for most of my colleagues) are limited exclusively to points of detail, generally quickly spotted by myself. They are simple 'incidents of the trail', of a purely 'local' nature and without any serious impact on the validity of the essential intuitions concerning the situation under examination. On the other hand, at the level of the ideas and the great guiding intuitions, it seems to me that my work is free of any “failures*[raté]"*, however incredible this may seem. It is this never failing certainty to apprehend at every moment, if not the ultimate <u>results</u> of an approach (which most often remain hidden from view), then at least the most fertile <u>directions</u> that offer themselves to lead me straight to the <u>essential</u> things - it is this certainty that had resurged in me Koestler's image of the "sleepwalkers".

[2] From the 1960s onwards, some of these publications were written in collaboration with colleagues (especially J. Dieudonné) and students.

given them when I had brought them out([1]). In the history of mathematics, I believe that I am the one who has introduced into our science the greatest number of new notions, and at the same time, the one who has been led, by this very fact, to invent the greatest number of new names, in order to express these notions with delicacy, and in a way that is as suggestive as I could.

These 'quantitative' details certainly provide only a rough apprehension of my work, missing what is really its soul, its life and its vigour. As I wrote earlier, the best thing I have contributed to mathematics are the new "points of view" that I was able to glimpse at the beginning, and then patiently draw out and develop to varying degrees. Like the notions I have just mentioned, these new points of view, introduced into a vast multiplicity of very different situations, are themselves virtually innumerable.

There are, nevertheless, some views that are broader than others, and which alone give rise to and encompass a multitude of partial views in a multitude of different particular situations. Such a viewpoint can also be rightly called a 'great idea'. By its own fecundity, such an idea gives rise to a teeming progeny, all of which inherit its fecundity, but most (if not all) of which are less far-reaching than the mother idea.

As for expressing a great idea, "saying" it, this is, more often than not, almost as delicate as its conception and its slow gestation in the person who conceived it - or to put it better, this laborious work of gestation and formation is precisely that which "expresses" the idea: the work that consists in patiently drawing it out, day after day, from the veils of mist that surround it at its birth, in order to gradually give it a tangible form, in a picture that becomes richer, firmer and more refined as the weeks, months and years go by. Simply naming the idea, by some striking formula, or by more or less technical key words, can be a matter of a few lines, or even a few pages - but there will be very few people who, without already knowing it well, will be able to understand this "name" and recognise in it a visage. And when the idea has reached full maturity, perhaps a hundred pages will suffice to express it, to the full satisfaction of the worker in whom it was born - yet it may also happen that ten thousand pages, worked on and thought over at length, will not suffice([2]).

And in either case, among those who, in order to absorb the idea, have got acquainted with the work that finally presents the idea in full bloom, like a spacious forest have grown there on a deserted heath - it is likely that many will see all these vigorous and slender trees and will make use of them (some to climb them, some to pull out beams and boards, and some to light fires in their fireplaces). But few will be able to see the forest...

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[1] The most imposing of these notions are reviewed in the Thematic Sketch, and in the accompanying History Commentary, which will be included in volume 4 of the Reflections. Some of the names were suggested to me by friends or students, such as the term "smooth morphism" (J. Dieudonné) or the panoply "site, stack, gerbe, lien", developed in Jean Giraud's thesis.

[2] By the time I left the mathematical scene in 1970, the totality of my publications (many of them in collaboration) on the central theme of schemes must have amounted to some ten thousand pages. This was, however, only a modest part of the large-scale programme I saw before me, concerning schemes. This programme was abandoned sine die as soon as I left, despite the fact that almost everything that had already been developed and published to be made available to all, immediately became part of the common heritage of notions and results commonly used as 'well known'.

The part of my programme on the schematic theme and its extensions and ramifications, which I had accomplished at the time of my departure, represents in itself the most vast work of foundations ever accomplished in the history of mathematics, and surely one of the most extensive in the history of Science.

8. The vision - or twelve themes for a harmony

Perhaps we can say that the "great idea" is the point of view that not only proves to be new and fecund, but introduces into science a new and vast theme that embodies it. And every science, when we understand it not as an instrument of power and domination, but as an adventure of knowledge*[connaissance]* of our species across the ages, is nothing but this harmony, more or less vast and more or less rich from one epoch to another, which unfolds over the course of generations and centuries, through the delicate counterpoint of all the themes appearing in turn, as if summoned*[appelés]* from the void, to join and intertwine within it.

Among the many new points of view that I have brought to light in mathematics, there are twelve, which, with hindsight, I would call "great ideas"([1]). To see my work as a mathematician, to "feel" it, is to see and "feel" at least some of these ideas, and these great themes that they introduce and which make up the fabric and soul of the work.

By the force of things, some of these ideas come out "greater" than others (which, thereby, are "smaller"). In other words, among these new themes, some are vaster than others, and some plunge deeper into the heart of the

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[1] Here, for the mathematician reader who might be curious, is the list of these twelve main*[maîtresses]* ideas, or "master*[maître]* themes" of my work (in chronological order of appearance)

1. Topological tensor products and nuclear spaces.
2. “Continuous" and "discrete" dualities (derived categories, "six operations”).
3. Riemann-Roch-Grothendieck yoga (K-Theory, relation to intersection theory).
4. Schemes.
5. Topos.
6. Etale and l-adic cohomology.
7. Motives and motivic Galois group (Grothendieck \tensor-categories).
8. Crystals and crystalline cohomology, yoga of “De Rham coefficients”, “Hodge coefficients”...
9. “Topological algebra”: \infty-stacks, derivators, cohomological formalism of topos, as an inspiration for a new homotopical algebra.
10. Tame topology.
11. Yoga of anabelian algebraic geometry, Galois–Teichmüller theory.
12. "Schematic" or "arithmetic" point of view for regular polyhedra and regular configurations of all kinds.

Apart from the first of these themes, an important part of which is part of my thesis (1953) and was developed in my functional analysis period between 1950 and 1955, the other eleven were developed during my period as a geometer, starting in 1955.

mystery of mathematical things[1]. There are three (and not the least in my eyes) which, having appeared only after my departure from the mathematical scene, are still in an embryonic state: "officially" they do not even exist, since no formal publication is there to serve as their birth certificate[2]. Of the nine themes that appeared before my departure, the last three, which I had left in a flourishing state, remain today in their infancy, for lack (after my departure) of loving hands to provide for the needs of these "orphans", left behind in a hostile world[3]. As for the other six themes, which reached full maturity in the two decades preceding my departure, it can be said (with one or two reservations[4]) that they had already become part of the common heritage by that time: especially among the geometricians, "everyone" nowadays intones them without even knowing it (as Mr. Jourdain did with his prose), all day long and at all times. They are part of the air we breathe when we "do geometry", or when we do arithmetic, algebra or analysis that is "geometric" in the slightest way.

These twelve great themes of my work are by no means isolated from each other. To me, they are part of a unity of spirit and purpose, present, like a common and persistent background note, throughout my 'written' and 'unwritten' work. And as I write these lines, I seem to find the same note again - like a call*[appel]*! - through

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[1] Among these themes, the vastest in scope seems to me to be that of topos, which provides the idea of a synthesis of algebraic geometry, topology and arithmetic. The vastest in the extent of developments to which it has given rise, is the theme of schemes. (See on this subject footnote (*) page 20 *[the previous note -Trans.]*.) It is this theme which provides the framework "par excellence" for eight others among these envisaged themes (namely, all the others excluding themes 1, 5 and 19), at the same time it provides the central notion for a thorough renewal of algebraic geometry, and of the algebraic-geometric language.

On the other hand, the first and last of the twelve themes seem to me to be of more modest dimensions than the others. However, as far as the last one is concerned, which introduces a new perspective in the very old theme of regular polyhedra and regular configurations, I doubt that the life of a mathematician who devotes himself to it would be enough to exhaust it. As for the first of all these themes, that of topological tensor products, it has played more of a role as a new ready-to-use tool than as a source of inspiration for further developments. Yet I still receive, up until the last few years, sporadic echoes of more or less recent work, resolving (twenty or thirty years later) some of the questions that I had left unsolved.

The most profound (to me) of these twelve themes are that of motives, and the closely related one of anabelian algebraic geometry and Galois-Teichmüller yoga.

From the point of view of the power of tools perfectly developed and honed by me, and in common use in various "leading-edge sectors" in research over the last two decades, it is the aspects of "schemes" and "étale and l-adic cohomology" that seem to me to be the most notable. For a well-informed mathematician, I think that from now on there can hardly be any doubt that the schematic tool, as well as that of l-adic cohomology which is derived from it, are part of the few great achievements of the century, which have come to nourish and renew our science during the last generations.

[2] The only 'semi-official' text in which these three themes are sketched out to any extent is the Sketch of a Programme, written in January 1984 on the occasion of an application for a secondment to the CNRS. This text (also mentioned in Introduction 3, "Compass and Luggage") will in principle be included in volume 4 of the Reflections.

[3] After the burial of these three orphans without fanfare, the day after my departure, two of them were exhumed with great fanfare and without any mention of the worker, one in 1981 and the other (given the flawless success of the operation) the following year.

[4] The ‘one or two things' are mostly about the Grothendieckian yoga of duality (derived categories and six operations), and that of the topos. This will be discussed in detail (among other things) in Parts II and IV of Harvest and Sowing (Burial (1) and (3)).

those three years of 'free', relentless and solitary work, at a time when I had not yet cared whether there were any mathematicians in the world apart from myself, so caught up was I then in the fascination of what was calling me...

This unity is not only the result of the mark of the same worker on the works that come out of his hands. These themes are linked to each other by innumerable connections, both delicate and obvious, just like the different themes themselves, each clearly recognisable, which unfold and intertwine in the same vast counterpoint - in a harmony that brings them together, carries them forward and gives each one a meaning, a movement and a fullness in which all the others participate. Each of the partial themes seems to be born of this vaster harmony and to be reborn from it again and again, rather than the harmony appearing as a "sum" or as a "result" of constituent themes that pre-exist it. And to tell the truth, I can't help but feel (a bit crazy no doubt...) that in a certain way it is indeed this harmony, not yet appeared but which surely already "existed", somewhere in the obscure bosom of things still to be born - that it is indeed this harmony which in turn gave rise to these themes which were to take on their full meaning only through it, and that it is also this harmony which already called to me in a low, pressing voice, in those years of ardent solitude, at the end of adolescence...

The fact remains that these twelve master-themes of my work are all, as if by secret predestination, part of the same symphony - or, to use a different image, they embody so many different "points of view", all contributing to the same vast <u>vision</u>.

This vision only began to emerge from the mists, to show recognisable outlines, around the years 1957, 58 - years of intense gestation([1]). Strangely perhaps, this vision was so close to me, so "obvious", that until a year ago([2]), I had not thought of giving it a name (Although one of my passions has been to constantly <u>name</u> the things that come to me, as a first means of apprehending them...) It is true that I cannot point to a particular

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[1] The year 1957 was the year in which I was led to bring out the '<u>Riemann-Roch</u>' theme (Grothendieck version) - which, overnight, made me a 'big star'. It was also the year of my mother's death, and thus the year of an important break in my life. It was one of the most intensely creative years of my life, and not only in mathematics. For twelve years all my energy had been invested in mathematical work. That year, I had the feeling that I had pretty much 'done*[fait le tour]*' with mathematical work, that it might be time to invest myself in something else. It was a need for inner renewal, visibly, that surfaced then, for the first time in my life. I thought at that time about becoming a writer, and for several months I stopped all mathematical activities. Finally, I decided that I would at least put the mathematical work I was already doing in black and white, taking a few months probably, or a year at the most...

The time was not yet ripe, no doubt, for the great leap. Nevertheless, once I had resumed my mathematical work, it took me back. It did not let go of me for another twelve years!

The year that followed this interlude (1958) is perhaps the most fruitful of all in my life as a mathematician. It was in this year that the two central themes of the new geometry bloomed, with the strong start of the <u>theory of schemes</u> (the subject of my exposé at the International Congress of Mathematicians in Edinburgh in the summer of that year), and the appearance of the notion of '<u>site</u>', a provisional technical version of the crucial notion of <u>topos</u>. With the benefit of almost thirty years' hindsight, I can now say that this was the year in which the vision of the new geometry was really born, in the wake of the two master tools of this geometry: the schemes (which represent a metamorphosis of the old notion of "algebraic variety"), and the topos (which represent an even more profound metamorphosis, of the notion of space).

[2] I first thought of naming this vision in the reflection of 4 December 1984, in the sub-note (no. 136.1) to the note "Yin the Servant (2) - or generosity" (ReS III, page 637).

moment, which could have been perceived as the moment of the appearance of this vision, or which I could recognise as such in retrospect. A new vision is such a vast thing, that its appearance cannot be situated at a particular moment, but must penetrate and take possession progressively over many years, if not over generations, of those who scrutinise and contemplate; as if new eyes had to be laboriously formed, behind the familiar eyes which they are called upon to replace little by little. And the vision is also too vast to be "grasped", as one would grasp the first notion that appeared at the turn of the road. This is why it is probably not surprising, in the end, that the thought of naming something so vast, and so close and so diffuse, appeared only in retrospect, once it had reached full maturity.

To tell the truth, until two years ago my relationship with mathematics was limited (apart from the teaching task) to doing it - to following an impulse that constantly pulled me forward, into an 'unknown' that constantly attracted me. The idea did not occur to me to stop in this momentum, to pause for even a moment, to look back and see perhaps a path that had been travelled, or even to situate a work that had been done. (Whether to situate it in my life, as something to which deep and long-ignored links continue to connect me; or also, to situate it in this collective adventure that is “mathematics").

Strangely enough, in order to bring me to finally "put down" and get reacquainted with this half-forgotten work, or to even think of giving a name to the vision that was its soul, I had to suddenly find myself confronted with the reality of a Burial of gigantic proportions: with the burial, through silence and derision, of both the vision and the worker in whom it was born...

9. Form and structure - or the voice of things

Without having planned it, this "foreword" ended up becoming a sort of formal presentation of my work, intended (above all) for the non-mathematician reader. I am already too involved to be able to backtrack any further, so all that remains is to complete the "presentations"! I would like to try to say at least a few words about the substance of these fabulous*[mirifiques]* "great ideas" (or “master-themes") that I have been hinting at*[miroiter]* in the previous pages, and about the nature of the famous "vision" into which these master ideas are supposed to converge. Without being able to use any technical language, I will probably only be able to convey an extremely vague image (if anything does indeed "get through"...)([1]).

[1] The fact that this image must remain "vague" does not prevent it from being faithful, or prevent it from genuinely rendering something of the essence of what is being examined (in this case, my work). Conversely, an image may be sharp, but it may well be distorted, and moreover, include only the incidental and miss the essential entirely. Also, if you "hang on" to what I see to say about my work (and surely then something of the image in me will indeed "get through"), you will be able to flatter yourself that you have grasped what is essential in my work better than perhaps any of my learned colleagues!

Traditionally, there are three types of "qualities" or "aspects" of things in the Universe, which are the object of mathematical reflection: these are number[1], magnitude*[grandeur]*, and form. They can also be called the "arithmetic" aspect, the "metric" (or "analytic") aspect, and the "geometric" aspect of things. In most situations studied in mathematics, these three aspects are present simultaneously and in close interaction. However, more often than not, there is a marked predominance of one of the three. It seems to me that for most mathematicians it is quite clear (to those who know them, or who are familiar with their work) what their basic temperament is, whether they are "arithmeticians", "analysts", or "geometers" - even though they might have many strings on their violin, and might work in every conceivable register and pitch.

My first and solitary reflections, on the theory of measure and integration, are unambiguously placed in the category of 'magnitude', or 'analysis'. And the same is true of the first of the new themes that I introduced into mathematics (which appears to me to be of lesser dimensions than the other eleven). That I entered mathematics through the "bias" of analysis seems to me to be due, not to my particular temperament, but to what can be called a "fortuitous circumstance": it is that the most enormous gap, to my mind fond of generality and rigour, in the teaching which was available to me at the lycée as well as at the university, concerned the "metric" or "analytical" aspect of things.

The year 1955 marked a crucial turning point in my mathematical work: the transition from 'analysis' to 'geometry'. I still remember this striking impression (totally subjective, of course), as if I were leaving the arid and rough steppes, to suddenly find myself in a sort of "promised land" of luxuriant riches, multiplying infinitely wherever the hand likes to lay upon, to pick*[cueillir]* or to dig*[fouiller]*... And this impression of overwhelming*[accablant]* wealth, beyond all measure[2], has only been confirmed and deepened over the years, right up to the present day.

That is to say that if there is one thing in mathematics which (probably since the very beginning) fascinates me more than any other, it is neither "the number", nor "the magnitude", but always the form. And among the thousand-and-one faces that form chooses to reveal itself to us, the one that has fascinated me more than any other and continues to fascinate me is the structure hidden in mathematical things.

The structure of a thing is by no means something we can 'invent'. We can only patiently, humbly uncover it - get to know it, 'discover' it. If there is inventiveness in this work, and if we sometimes work as blacksmiths or indefatigable builders, it is by no means to 'shape' or 'build' the 'structures'. These structures did not wait for us to be, to be exactly what they are! But it is to express, as faithfully as we can, these things that we are in the process of discovering and probing, and this structure that is reluctant to surrender itself, that we are groping

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[1] It is understood here that we are talking about "numbers" known as "natural integers" 0, 1, 2, 3, etc., or (at the very least) numbers (such as fractional numbers) which can be expressed using these numbers by operations of an elementary nature. These numbers do not lend themselves, like the "real numbers", to measuring a quantity that can vary continuously, such as the distance between two variable points on a line, in a plane or in space.

[2] I have used the word combination "overwhelming*[accablant]*, beyond all measure" to render the German expression "überwältigend" and its English equivalent "overwhelming" as best I can. In the previous sentence, the (inadequate) expression "striking impression" is also to be understood with this nuance: when the impressions and feelings aroused in us by the confrontation with an extraordinary splendour, grandeur or beauty suddenly overwhelm us, to the point that any attempt to express what we feel seems to be vanquished from the start.

for, by means of a language that is perhaps still in its infancy, to establish*[cerner]*. Thus we are led to constantly 'invent' the language capable of expressing in ever greater detail the intimate structure of the mathematical thing, and to 'construct' with the help of this language, as we go along and from scratch, the 'theories' that are supposed to account for what has been apprehended and seen. There is a continuous, uninterrupted back-and-forth movement between the apprehension of things and the expression of what is apprehended, by means of a language that is refined and recreated as the work progresses, under the constant pressure of immediate need.

As the reader will no doubt have guessed, these 'theories', 'constructed from scratch', are none other than the 'beautiful houses' mentioned earlier: those we inherit from our predecessors, and those we are led to build with our own hands, by calling and listening to things. And if I spoke earlier of the "inventiveness" (or imagination) of the builder or the blacksmith, I should add that what makes it the soul and the secret nerve is by no means the superbness of the person who says: "I want this, and not that!" and who takes pleasure in deciding as he pleases; like a poor architect who would have his plans ready in his head, before having seen and felt the terrain, and having probed its possibilities and requirements. The quality of the researcher's inventiveness and imagination, lies in the quality of his attention, in listening to the voice of things. For the things of the Universe never tire of speaking for themselves and revealing themselves to those who care to hear. And the most beautiful house, the one in which the worker's love appears, is not the one that is bigger or higher than others. The beautiful house is the one that faithfully reflects the hidden structure and beauty of things.

10. The new geometry - or the nuptials of number and magnitude

But here I am diverging again - I proposed to speak of the master-themes, coming together in the same mother-vision, like so many rivers returning to the Sea of which they are the sons...

This vast unifying vision can be described as a new geometry. It is the one that Kronecker dreamed of in the last century([1]). But the reality (which a bold dream sometimes makes us sense or glimpse, and encourages us to discover...) is always richer and more resonant than even the boldest or deepest dream. Surely, more than one of the aspects of this new geometry (if not for all of them), no one, on the day before it appeared, would have thought of it - and the worker himself no more than the others.

We can say that "number" is capable of grasping the structure of "discontinuous" or "discrete" aggregates: systems, usually finite, made up of "elements" or "objects", so to speak, isolated from one another, without some principle of "continuous passage" from one to another. "Magnitude", on the other hand, is the quality par

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[1] I only know about this 'Kronecker's dream' from hearsay, when someone (perhaps it was John Tate) told me that I was making it come true. In the education I received from my elders, historical references were rare, and I was nourished, not by reading ancient or even contemporary authors, but mainly by communicating, verbally or through letters, with other mathematicians, starting with my elders. The main, perhaps even the only external inspiration for the sudden and vigorous start of the theory of schemes in 1958, was Serre's paper, well known under the acronym FAC ("Faisceaux algébriques cohérents"), published a few years earlier. This one aside, my main inspiration in the further development of the theory was to be found to flow from itself, and to be renewed over the years, by the sole requirements of simplicity and internal coherence, in an effort to account in this new context for what was "well known" in algebraic geometry (and which I was assimilating as it was being transformed in my hands), and for what this "known" made me sense.

excellence, capable of "continuous variation"; hence it is capable of grasping continuous structures and phenomena: movements, spaces, "varieties" of all kinds, force fields etc. Thus, arithmetic appears (roughly speaking) as the science of discrete structures, and analysis as the science of continuous structures.

As for geometry, it can be said that for more than two thousand years that it has existed in the form of a science in the modern sense of the word, it has been "straddling" these two types of structures, the "discrete" and the "continuous"([1]). For a long time, moreover, there was no real "divorce" between two geometries that seem to be of different kinds, one discrete, the other continuous. Rather, there were two different points of view in the investigation of the same geometric figures: one emphasising "discrete" properties (and in particular, numerical and combinatorial properties), the other "continuous" properties (such as position in the surrounding space, or "magnitude" measured in terms of mutual distances of its points, etc).

It was at the end of the last century that a divorce appeared, with the appearance and development of what has sometimes been called "abstract (algebraic) geometry". Roughly speaking, this consisted in introducing, for each prime number p, a (algebraic) geometry "of characteristic p", modelled on the (continuous) model of the (algebraic) geometry inherited from the previous centuries, but in a context, however, which appeared to be fundamentally "discontinuous" and "discrete". These new geometric objects have become increasingly important since the beginning of the century, and, especially, in view of their close relationship with arithmetic, the science par excellence of discrete structure. It would seem to be one of the guiding ideas in the work of André Weil([2]), perhaps even the main thrust (which remained more or less unstated in his written work, as it should), that (algebraic) geometry, and especially the "discrete" geometries associated with the various prime numbers, were to provide the key to a large-scale renewal of arithmetic. It was in this spirit that, in 1949, he came up with the famous "Weil conjectures". These were absolutely astonishing conjectures, indeed, which made it possible to foresee, for these new "varieties" (or "spaces") of a discrete nature, the possibility of certain types of constructions and arguments([3]) which until then had only seemed conceivable only within the framework o the "spaces" considered worthy of the name by analysts - namely, the so-called "topological" spaces (where the notion of continuous variation applies).

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[1] In fact, traditionally it was the "continuous" aspect that was the focus of the geometer's attention, while the "discrete" properties, and in particular the numerical and combinatorial properties, were passed over in silence or treated under the leg. It was with wonder that I discovered, about ten years ago, the richness of the combinatorial theory of the icosahedron, whereas this theme is not even touched upon (and probably not even seen) in Klein's classic book on the icosahedron. I see another striking sign of this (two thousand year old) neglect by geometers of the discrete structures that spontaneously introduce themselves into geometry: it is that the notion of group (of symmetries, in particular), which only appeared in the last century, and that, moreover, it was first introduced (by Evariste Galois) in a context that was not then considered to be part of "geometry. It is true that even today, many algebraists have still not understood that Galois' theory is indeed, in its essence, a "geometrical" vision, renewing our understanding of the so-called "arithmetical" phenomena...

[2] André Weil, a French mathematician who emigrated to the United States, is one of the 'founding members' of the 'Bourbaki group', which will be discussed at length in the first part of Harvest and Sowing (as well as Weil himself, occasionally).

[3] (For the benefit of the mathematician reader.) These are "constructions and arguments" related to the cohomological theory of differentiable or complex varieties, and in particular those involving Lefschetz fixed point formula, and Hodge theory.

The new geometry can be considered, above all, as a synthesis between these two worlds, which until then had been adjoining and closely interdependent, but yet separate: the "arithmetical" world, in which the (so-called) "spaces" without a principle of continuity live, and the world of continuous magnitude, in which "spaces" in the proper sense of the term live, accessible to the analyst's means and (for this very reason) accepted by him as worthy of living in the mathematical city. In the new vision, these two formerly separate worlds now form a single whole.

The first embryo of this vision of an "arithmetic geometry" (as I propose to call this new geometry) can be found in Weil's conjectures. In the development of some of my main themes([1]), these conjectures remained my main source of inspiration, throughout the years between 1958 and 1969. Even before me, moreover, Oscar Zariski on the one hand, and Jean-Pierre Serre on the other, had developed certain "topological" methods for the faithless-and-lawless-spaces of "abstract" algebraic geometry, inspired by those that had previously been used for everyone else's "well-tinted*[bon teint]* spaces"([2]), and their ideas, of course, played an important role in my first steps towards building arithmetic geometry; more, in fact, as starting points and tools (which I had to reshape more or less from scratch, for the needs of a much larger context), than as a source of inspiration that would have continued to nourish my dreams and projects, over the months and years. In any case, it was clear from the outset that, even reshaped, these tools fell far short of what was required to take even the very first steps in the direction of the fantastic conjectures.

11. The magic fan*[éventail]* - or the innocence

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[1] These are the four 'middle' themes (no. 5 to 8), namely those of topos, étale and l-adic cohomology, motives, and (to a lesser extent) crystals. I brought out these themes one after another between 1958 and 1966.

[2] (For the mathematical reader.) Zariski's main contribution in this direction seems to me to be the introduction of "Zariski topology" (which later became an essential tool for Serre in FAC), and his "principle of connectedness" and what he called his "theory of holomorphic functions" - which became in his hands the theory of formal schemes, and the "comparison theorems" between the formal and the algebraic (with, as a second source of inspiration, Serre's fundamental article GAGA). As for Serre's contribution to which I allude in the text, it is of course, above all, the introduction by him, in abstract algebraic geometry, of the point of view of sheaves (introduced by Jean Leray a dozen years earlier, in a quite different contest), in this other fundamental article already quoted FAC ("Faisceaux algébriques cohérents”).

In the light of these "reminders", if I were to name the immediate "ancestors" of the new geometrical vision, the names of Oscar Zariski, André Weil, Jean Leray and Jean-Pierre Serre immediately come to mind. Among them, Serre played a special role, since it was mainly through him that I became aware not only of his own ideas, but also of the ideas of Zariski, Weil and Leray, which had to play a role in the emergence and development of the new geometry.

The two crucial ideas in the start-up and development of the new geometry were that of scheme and that of topos. Appearing more or less simultaneously and in close symbiosis with each other([1]), they were like a single and same motor nerve in the spectacular rise of the new geometry, and this from the very year of their appearance. To conclude this overview of my work, it remains for me to say a few words about at least these two ideas.

The notion of scheme is the most natural, the most "obvious" imaginable, for encompassing in a single notion the infinite series of notions of (algebraic) "variety" that we used to deal with before (one such notion for each prime number([2])...). Moreover, one and the same "scheme" (or "variety" in the new style) gives rise, for each prime number p, to a well-defined "(algebraic) variety of characteristic p". The collection of these different varieties of the different characteristics can then be visualised as a kind of "(infinite) fan of varieties" (one for each characteristic). The "scheme" is this magical fan, which links together, as so many different "branches", its "avatars" or "incarnations" of all possible characteristics. In this way, it provides an effective "principle of passage" to link together "varieties", stemming from geometries which until then had appeared more or less isolated, cut off from each other. Now they find themselves encompassed in a common "geometry" and linked by it. It could be called schematic geometry, the first draft of this "arithmetic geometry" into which it was to develop in the following years.

The very idea of a scheme is childishly simple - so simple, so humble, that no one before me had thought of bending so low. So 'silly', in fact, that for years, despite the obvious, many of my learned colleagues thought it was really 'not serious'! It took me months of hard and solitary work to convince myself that it really did "work*[marchait]*" - that the new language, so silly, that I was incorrigibly naive enough to insist on testing, was really adequate to grasp, in a new light and with a new finesse, and in a common framework from then on, some of the very first geometrical intuitions attached to the previous "geometries of characteristic p". It was the kind of exercise, judged in advance to be silly and hopeless by any "well-informed" person, that I was probably the only one, among all my colleagues and friends, to ever have the idea of putting it on top*[mettre en tête]*, and indeed (driven by a secret demon...) to carry it through to a successful conclusion against all odds!

Rather than being distracted by the ruling consensus around me about what is 'serious' and what is not, I simply trusted, as in the past, the humble voice of things, and that in me which knows how to listen. The reward was immediate, and beyond all expectations. In the course of these few months, without even "doing it on purpose", I had put my finger on some powerful and unsuspected tools. They allowed me not only to rediscover (as if playing) old results, reputed to be difficult, in a more penetrating light and to go beyond them, but also to finally

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[1] This start, which takes place in 1958, is mentioned in the note (*) page 23*[first note on page 25 -Trans.]*. The notion of site or 'Grothendieck topology' (a provisional version of that of topos) appeared in the immediate wake of the notion of scheme. It is this in turn that provides the new language of 'localisation' or 'descent', used at every step in the development of the schematic theme and tool.

The more intrinsic and geometric notion of topos, which at first remained implicit during the following years, emerged especially from 1963, with the development of étale cohomology, and gradually imposed itself on me as the most fundamental notion.

[2] It should also be included in this series the case p = 1, corresponding to algebraic varieties “of zero characteristic".

approach and solve problems of "p-characteristic geometry" which until then had appeared to be out of reach by all the means then known([1]).

In our knowledge of the things of the Universe (be they mathematical or otherwise), the renewing power in us is none other than innocence. It is the original innocence that we were all born with and which lies within each of us, which is often the object of our contempt and our most secret fears. It alone unites the humility and boldness that allow us to penetrate to the heart of things, and that allow us to let things penetrate to us and to be imbued*[imprégner]* with them.

This power is by no means the privilege of extraordinary "gifts" - of (let us say) extraordinary brain power to assimilate and handle, with dexterity and ease, an impressive mass of known facts, ideas and techniques. Such gifts are certainly precious "beyond all measures", worthy of envy for those who (like me) were not so blessed at birth.

It is not these gifts, however, nor even the most ardent ambition, served by an unwavering will, that make us cross these "invisible and imperious circles" that enclose our Universe. Only innocence crosses them, without knowing or caring, in those moments when we find ourselves alone, listening to things, intensely absorbed in a child's game...

12. The topology - or the surveying of mists

The innovative idea of the "scheme", as we have just seen, is that of linking together the different "geometries" associated with the different prime numbers (or different "characteristics"). These geometries, however, were still each of an essentially "discrete" or "discontinuous" nature, in contrast to the traditional geometry bequeathed by past centuries (and dating back to Euclid). The new ideas introduced by Zariski and Serre restored, to a certain extent, a "dimension" of continuity to these geometries, inherited immediately by the "schematic geometry" that had just appeared, in order to unite them. But as far as the "fantastic conjectures" (of Weil) were concerned, we were very far from the mark. These 'Zariski topologies' were, from this point of view, so crude that it was almost as if they had remained at the stage of 'discrete aggregates'. What was missing, clearly, was some new principle, which would make it possible to link these geometric objects (or "varieties", or "schemes") to the usual (topological) "spaces", or "good*[bon teint]*" ones; those, say, whose "points" appear to be clearly separated from each other, whereas in the lawless-spaces introduced by Zariski, the points have an unfortunate tendency to cluster together...

It was certainly the appearance of such a 'new principle', and nothing less, that could bring about the 'marriage of number and magnitude' or of the 'geometry of the discontinuous' with that of the 'continuous', of which a first premonition*[pressentiment]* emerged from Weil's conjectures.

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[1] The account of this "strong start" of the theory of schemes is the subject of my exposé at the International Congress of Mathematicians in Edinburgh in 1958. The text of this exposé seems to me to be one of the best introductions to the point of view of schemes, of such a nature (perhaps) as to motivate a geometrician reader to familiarise himself as best he can with the imposing (later) treatise "Eléments de Géométrie Algébrique", which presents in a detailed manner (and without making grace of any technical details) the new foundations and techniques of algebraic geometry.

The notion of "space" is probably one of the oldest in mathematics. It is so fundamental to our "geometric" understanding of the world that it has remained more or less unspoken for over two millennia. It is only in the course of the last century that this notion has, progressively, detached itself from the tyrannical grip of the immediate perception (of one and the same "space" that surrounds us), and from its traditional ("Euclidean") theorisation, to acquire its own autonomy and dynamics. Nowadays, it is one of the few notions most universally and commonly used in mathematics, familiar to every mathematician without exception. A protean notion, if ever there was one, with a hundred and one thousand faces, depending on the type of structures incorporated into these spaces, from the richest of all (such as the venerable "Euclidean" structures, or the "affine" and "projective" structures, or even the "algebraic" structures of the "varieties" of the same name, which generalise them and make them more flexible), to the most stripped-down: those where any "quantitative" piece of information whatsoever seems to have disappeared without return, and where only the qualitative quintessence of the notion of "proximity" or that of "limit"([1]), and the most elusive version of the intuition of form (called "topological") remain. The most stripped-down of all these notions, the one that until now, over the past half-century, had taken the place of a kind of vast common conceptual bosom to encompass all the others, was that of topological space. The study of these spaces constitutes one of the most fascinating and lively branches of geometry: topology.

However elusive this structure of "pure quality" embodied by a "space" (called "topological") may seem at first sight, in the absence of any data of a quantitative nature (such as the distance between two points, in particular) that would allow us to cling to some familiar intuition of "largeness*[grandeur]*" or "smallness", we have nevertheless managed, in the course of the past century, to finely define these spaces in the tight and flexible meshes of a language carefully "tailored to pieces*[taillé sur pièces]*". Even better, we have nonetheless invented and fabricated from scratch some sort of "meters" or "toises" to be used, against all odds, to attach some sort of "measures" (called "topological invariants") to these sprawling "spaces" which seemed to evade, like imperceptible mists, any attempt at measurement. It is true that most of these invariants, and the most essential ones, are of a more subtle nature than a simple "number" or "magnitude" - they are rather themselves more or less delicate mathematical structures, attached (by means of more or less sophisticated constructions) to the space under consideration. One of the oldest and most crucial of these invariants, introduced as early as the last century (by the Italian mathematician Betti), is formed by the various "groups" (or "spaces") called

[1] Speaking of the notion of "limit", it is mainly the notion of "crossing the limit" that I have in mind here, rather than the notion (more familiar to the non-mathematician) of "frontier".

“cohomology groups (or spaces)“, associated with the space([1]). It is these invariants which intervene (mostly "between the lines", admittedly) in Weil's conjectures, that give them their deep "raison d'être" and that (for me at least, "put in the bath*[mis dans le bain]*" by Serre's explanations) give them their full meaning. But the possibility of associating such invariants to the "abstract" algebraic varieties which intervene in these conjectures, so as to meet the very precise desiderata required for the needs of that cause - that was a simple hope. I doubt that, apart from Serre and myself, anybody else (not even, and especially, André Weil himself!([2])) really believed in it...

Shortly before, our conception of these cohomology invariants had been enriched and profoundly renewed by the work of Jean Leray (pursued in captivity in Germany, during the war, in the first half of the forties). The

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[1] In fact, the invariants introduced by Betti were homology invariants. Cohomology is a more or less equivalent, "dual" version, introduced much later. This aspect has acquired a pre-eminence over the initial "homological" aspect, especially (probably*[sans doute]*) following the introduction, by Jean Leray, of the sheaf point of view, discussed below. From a technical point of view, it can be said that a large part of my work as a geometer consisted in bringing out, and developing to a greater or lesser extent, the cohomological theories that were lacking for spaces and varieties of all kinds, and above all, for "algebraic varieties" and schemes. Along the way, I was also led to reinterpret the traditional homological invariants in cohomological terms, and thus to see them in a completely new light.

There are many other "topological invariants" that have been introduced by topologists, to capture this or that type of properties of topological spaces. Apart from the "dimension" of a space, and the (co)homological invariants, the first other invariants are the "homotopy groups". I introduced another one in 1957, the (so-called "Grothendieck") group K(X), which immediately gained great popularity*[fortune]*, and whose importance (both in topology and in arithmetic) never ceases to be confirmed.

A host of new invariants, of a more subtle nature than the invariants currently known and used, but which I feel to be fundamental, are foreseen in my programme of 'tame topology' (a very rough outline of which can be found in the 'Sketch of a Programme', to be published in volume 4 of Reflections). This programme is based on the notion of "tame theory" or "tame space", which constitutes, somewhat like that of topos, a (second) "metamorphosis of the notion of space". It is much more obvious (it seems to me) and less profound than the latter. I foresee that its immediate repercussions on topology "proper" will however be much more striking, and that it will transform the "trade" of the geometric topologist from top to bottom, by a profound transformation of the conceptual context in which one works. (As it was also the case in algebraic geometry with the introduction of the point of view of the schemes.) I sent my "Sketch" to several of my old friends and illustrious topologists, but it does not seem to have interested any of them...

[2] Paradoxically, Weil had a tenacious, seemingly visceral 'block' against cohomological formalism - even though it was largely his famous conjectures that inspired the development of the great cohomological theories in algebraic geometry, from the 1955s onwards (with Serre giving the kick-off, with his seminal paper FAC, already mentioned in a previous footnote).

It seems to me that this "block" is part of Weil's general aversion to all "big stuff*[gros fourbis]*", to anything resembling formalism (when this cannot be summed up in a few pages), or to a "construction" that is in any way interwoven. He had nothing of the "builder" in him, certainly, and it was clearly against his will that he was forced, during the thirties, to develop the first foundations of 'abstract' algebraic geometry, which (given these provisions*[vu ces dispositions]*) proved to be a veritable 'Procrustean bed' for the user.

I don't know if he resented me for going beyond that, and for investing myself in building the vast residences that allowed the dreams of a Kronecker and his own to be embodied in a language and in delicate and effective tools. Still, at no time did he give me a word of comment about the work he saw me engaged in, or that which was already done. Nor did I get any response to Harvest and Sowing, which I had sent him over three months ago, with a warm dedication from my own hands.

essential new idea was that of an (abelian) sheaf on a space, to which Leray associated a series of corresponding "cohomology groups" (said to have coefficients in this sheaf).It was as if the good old standard "cohomological metre" which we had until now for "surveying" a space had suddenly been multiplied into an unimaginably large multitude of new "metres" of all imaginable sizes, shapes and substances, each one intimately adapted to the space in question, and each one provides us with perfectly precise information about the space, which it alone can give us. This was the key idea in a profound transformation in our approach to spaces of all kinds, and surely one of the most crucial ideas to emerge in this century. Thanks especially to the later work of Jean-Pierre Serre, Leray's ideas had as their first fruits, already during the decade following their appearance, an impressive revival in the theory of topological spaces (and in particular, of their so-called "homotopy" invariants, closely linked to cohomology), and another revival, no less crucial, of the so-called "abstract" algebraic geometry (with Serre's fundamental article "FAC", published in 1955). My own work in geometry, from 1955 onwards, is in continuity with Serre's work, and thus also with Leray's innovative ideas.

13. The topos - or the double bed

The point of view and the language of sheaves introduced by Leray led us to look at "spaces" and "varieties" of all kinds in a new light. They did not, however, touch on the very notion of space, but merely making us apprehend more finely, with new eyes, these traditional "spaces", which were already familiar to all. However, it turned out that this notion of space is inadequate to account for the most essential "topological invariants" that express the "form" of "abstract" algebraic varieties (such as those to which Weil's conjectures apply), or even that of general "schemes" (generalising the old varieties). For the expected "marriages" of "number and magnitude", it was like a decidedly narrow bed, where only one of the future spouses (that is to say, the bride) could at best find a place to nestle, but never both at the same time! The "new principle" that remained to be found, in order to consummate the marriage promised by auspicious fairies, was none other than this spacious "bed" that the future spouses lacked, without anyone having noticed it until then...

This "double bed" appeared (as if by a magic wand...) with the idea of the topos. This idea encompasses, in a common topological intuition, both the traditional (topological) spaces, embodying the world of continuous magnitude, and the (so-called) "spaces" (or "varieties") of the unrepentant*[impénitents]* abstract algebraic geometers, as well as innumerable other types of structures, which until then had seemed irremediably riveted to the "arithmetical world" of "discontinuous" or "discrete" aggregates.

It is the point of view of the sheaves that has been the silent and sure guide, the efficient (and by no means secret) key, leading without procrastination or detours to the nuptial chamber with its vast conjugal bed. A bed so vast indeed (like a vast and peaceful river of great depth...), that

"all the king's horses
could drink together... “
[“tous les chevaux du roi
y pourraient boire ensemble. . . ”]

- as an old tune tells us, which surely you too must have sung, or at least heard sung. And he who was the first to sing it felt the secret beauty and the peaceful strength of the topos better than any of my learned students and friends of yesteryear...

The key has been the same, both in the initial and provisional approach (via the very convenient, but not intrinsic notion of 'site'), and in that of the topos. It is the idea of the topos that I would now like to try to describe.

Let us consider the set formed by <u>all</u> the sheaves on a given (topological) space, or, if you like, this prodigious arsenal formed by <u>all</u> these "metres" used to survey it([1]). We consider this "set" or "arsenal" as provided with its most obvious structure, which appears, so to speak, "at sight of the nose*[à vue de nez]*"; namely, a so-called "category" structure. (The non-mathematician reader need not worry about not knowing the technical meaning of this term. He will not need it for what follows). It is this sort of "surveying superstructure", called "category of sheaves" (over the space under consideration), which will henceforth be considered as "embodying" what is most essential to space. This is indeed lawful (for "mathematical common sense"), because it turns out that one can "reconstitute" a topological space([2]) from scratch in terms of this associated "category of sheaves" (or arsenal of surveying). (To check this is a simple exercise - once the question has been asked, of course...) This is all we need to be sure that (if it suits us for some reason) we can now 'forget' the initial space, and only retain and use the associated 'category' (or 'arsenal'), which will be seen as the most adequate embodiment of the 'topological' (or 'spatial') structure we are trying to express.

As is so often in mathematics, we have succeeded here (thanks to the crucial idea of "sheaf", or "cohomological metre") in expressing a certain notion (that of "space" in this case) in terms of another (that of “category"). Each time, the discovery of such a <u>translation</u> of a notion (expressing a certain type of situations) in terms of another (corresponding to another type of situations), enriches our understanding of both notions, through the unexpected confluence of specific intuitions that pertain to one or the other. Thus, a situation of a "topological" nature (embodied by a given space) is here translated by a situation of an "algebraic" nature (embodied by a "category"); or, if one wishes, the "continuum" embodied by space, is "translated" or "expressed" by the category structure, of an "algebraic" nature (and until then perceived as being of an essentially "discontinuous" or "discrete" nature).

But here, there is more. The first of these notions, that of space, appeared to us as a sort of "maximal" notion - a notion already so general, that it is hard to imagine how to find an extension that remains "reasonable". On the other hand, it turns out that on the other side of the mirror([3]), these "categories" (or "arsenals") that we come across, starting from topological spaces, are of a very particular nature. They enjoy a set of strongly typed

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1 (For the mathematician) Actually, we are talking about sheaves of <u>sets</u>, not <u>abelian</u> sheaves, introduced by Leray as the most general coefficients to form "cohomology groups”. I believe I was the first to work systematically with sheaves of sets (from 1955, in my paper "A general theory of fibre spaces with sheaf structure" at Kansas University).

2 (For the mathematician) Strictly speaking, this is only true for so-called "sober" spaces. However, these include almost all commonly encountered spaces, and in particular all the "separated" spaces dear to analysts.

3 The 'mirror' in question here, as in Alice in Wonderland, is the one that gives as an "image" of a space, placed in front of it, the associated "category", considered as a kind of "double" of the space, "on the other side of the mirror".

properties[1], which make them look like a sort of "pastiche" of the simplest imaginable kind - the one obtained by starting from a space reduced to a single point. That said, a "new style space" (or topos), generalising traditional topological spaces, will be described simply as a "category" which, without necessarily coming from an ordinary space, nevertheless has all those good properties (explicitly designated once and for all, of course) of such a "category of sheaves".

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This is the new idea. Its appearance can be seen as a consequence of this observation, in fact almost childish, that what really counts in a topological space are not its "points" or its subsets of points[2], and the relations of proximity etc. between them, but that it is the sheaves on this space, and the category they form. I have, in short, only taken Leray's initial idea to its ultimate consequence - and thereby taken the step forward*[franchir le pas]*.

Like the idea of sheaves (due to Leray), or that of schemes, like any "great idea" that shakes an inveterate vision of things, the idea of topos is disconcerting because of its naturalness, its "obviousness", its simplicity (bordering on the naïve or simplistic, or even "silly", as one might say) - because of that particular quality that so often makes us exclaim: "Oh, that's all it is!", with a tone of half-disappointment, half-envy; with, perhaps, in addition, the subtext of "eccentric", of "not serious", which we often reserve for everything that is disconcerting because of an excess of unexpected simplicity. Which reminds us, perhaps, of the long-buried and disowned days of our childhood...

14. Mutation of the notion of space - or the breath and the faith

The notion of scheme constitutes a vast extension of the notion of "algebraic variety", and as such it has renewed from top to bottom the algebraic geometry bequeathed by my predecessors. That of topos constitutes an unsuspected extension, or rather, a metamorphosis of the notion of space. In this way, it holds the promise of a similar renewal of topology, and beyond that, of geometry. As of now, indeed, it has played a crucial role in the rise of the new geometry (especially through the l-adic and crystalline cohomological themes that have emerged from it, and through them, in the proof of Weil's conjectures). Like its elder (and almost twin) sister, it possesses two complementary characters essential for any fertile generalisation, as follows.

[1] (For the mathematician) These are mainly properties that I have introduced into category theory under the name of 'accuracy properties[propriétés d'exactitude]' (together with the modern categorical notion of general inductive and projective "limits"). See "Sur quelques points d'algèbre homologique", *Tohoku math. journal*, 1957 (pp. 119-221).

[2] Thus, one can build very "big" topos, which have only one "point", or even no "points" at all!

Firstly, the new notion is not too broad, in the sense that in the new "spaces" (called "topos", so as not to offend delicate ears([1])), the most essential "geometric" intuitions and constructions([2]), familiar from the good old spaces of yesteryear, can be transposed in a more or less obvious way. In other words, the whole rich range of mental images and associations, notions and at least some of the techniques, which previously remained restricted to old-style objects, are available for the new objects.

And secondly, the new notion is at the same time broad enough to encompass a host of situations which, until now, were not considered to give rise to intuitions of a "topological-geometric" nature - intuitions which, in the past, had been reserved solely for ordinary topological spaces (and for good reason...).

The crucial thing here, from the point of view of Weil's conjectures, is that the new notion is indeed vast enough to allow us to associate with any "scheme" such a "generalised space" or "topos" (called the "étale topos" of the scheme under consideration). Some "cohomological invariants" of this topos (all that "silly" stuff!) seemed to have a good chance to provide "what was needed" to make sense of these conjectures, and (who knows!) perhaps to provide the means to prove them.

It is in these pages that I am writing that, for the first time in my life as a mathematician, I take the liberty of evoking (if only to myself) all the master-themes and the great guiding ideas in my mathematical work. This leads me to a better appreciation of the place and scope of each of these themes, and of the 'points of view' they embody, in the great geometrical vision that unites them and from which they stem. It is through this work that the two innovative ideas central to the first and powerful rise of the new geometry have come to light: the idea of schemes and that of topos.

It is the second of these ideas, that of the topos, which now appears to me as the more profound of the two. If, by any chance, towards the end of the fifties, I had not rolled up my sleeves, to obstinately develop day after day, throughout twelve long years, a 'schematic tool' of perfect delicacy and power - it would seem almost unthinkable to me, however, that in the ten or twenty years that followed, others than myself could have in the long run prevented themselves from introducing at the end of the ends (albeit unwillingly...) the notion that was obviously necessary, and to erect at least a few dilapidated "prefab" shacks, in the absence of the spacious and comfortable residences that I had the heart to assemble stone by stone and to build with my own hands. On the other hand, I can't think of anyone else on the mathematical scene, over the past three decades, who could have had the naivety, or the innocence, to take (in my place) that other crucial step of all, introducing the so childish idea of topos (or even that of "sites"). And, even supposing that this idea had already been graciously provided, and with it the modest promise it seemed to hold - I can think of no one else, either among my old friends or among my students, who would have had the impetus*[souffle]*, and above all the faith, to bring this humble

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[1] The name 'topos' was chosen (in association with 'topology', or 'topological') to suggest that it is the 'object par excellence' to which topological intuition applies. By the rich cloud of mental images that this name elicits, it should be regarded as more or less equivalent to the term (topological) 'space', simply with a greater emphasis on the 'topological' specificity of the notion. (Thus, there are "vector spaces", but no "vector topos" for the time being*[jusqu'à nouvel ordre]*!) It is necessary to keep the two expressions together, each with its own specificity.

[2] Among these "constructions", there is in particular that of all the familiar "topological invariants", including cohomological invariants. For the latter, I had done everything necessary in the article already quoted ("Tohoku" 1955), to be able to give them a meaning for any "topos".

idea[1] to fruition (so derisory in appearance, while the goal seemed infinitely distant...): from its early beginnings to the full maturity of the "mastery of étale cohomology", into which it has come to be embodied in my hands, in the years that followed.

15. All the horses of the king...

Yes, the river is deep, and vast and peaceful are the waters of my childhood, in a kingdom I thought I had left long ago. All the king's horses could drink from them at ease and to their content*[tout leur saoûl]*, without exhausting them! They come from the glaciers, fiery as those distant snows, and they have the softness of the clay of the plains. I have just spoken of one of these horses, which a child had led to drink and which drank its fill, at length. And I saw another one coming to drink for a while, following the trail of the same kid, for all I know - but that one didn't last long. Someone must have chased him away. And that's it, I suppose. Yet I see countless herds of thirsty horses roaming on the plain - and only this morning their neighing roused me from bed, at an ungodly hour, as I am approaching my sixtieth birthday and love tranquillity. There was nothing I could do, I had to get up. It pains me to see them, in the state of emaciated nags, although there is no lack of good water, or green pastures. But it seems as if a malevolent spell has been cast on this land that I had known to be welcoming, and condemned access to these generous waters. Or maybe it's a trick by the local horse traders, to bring down the prices, who knows? Or maybe it's a country where there are no more children to lead the horses to water, and where the horses are thirsty, for lack of a kid who finds the path that leads to the river...

16. The motives - or the heart within the heart

The theme of the topos is derived from that of the schemes in the same year that the schemes appeared - but in scope it goes far beyond the mother-theme. It is the theme of the topos, and not that of the schemes, which is this "bed", or this "deep river", where geometry and algebra, topology and arithmetic, mathematical logic and category theory, the world of the continuous and that of "discontinuous" or "discrete" structures, come together. If the theme of schemes is like the <u>heart</u> of the new geometry, the theme of the topos is its envelope, or <u>residence</u>. It is what I have conceived of as the vastest, in order to grasp with finesse, through the same language rich in geometric resonances, an "essence" common to situations that are the most distant from one another, coming from this region or that of the vast universe of mathematical things.

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[1] (For the interest of the mathematician reader.) When I speak of "bringing this humble idea to fruition", I am referring to the idea of étale cohomology as an approach to Weil's conjectures. It was inspired by this idea that I had discovered the notion of site in 1958, and that this notion (or the closely related notion of topos), and the étale cohomology formalism, were developed between 1962 and 1966 under my impetus (with the assistance of a few collaborators who will be mentioned later).

When I speak of " impetus " and " faith ", these are qualities of a "non-technical" nature, and which here appear to me as the essential qualities. On another level, I could also add what I would call "cohomological flair", that is to say the kind of flair that had developed in me for the construction of cohomological theories. I thought I was communicating it to my cohomology students. With the benefit of seventeen years' hindsight after my departure from the mathematical world, I can see that it has not been retained in any of them.

This theme of the topos is very far from enjoying the same fortune as that of the schemes. I will express myself on this subject on various occasions in Harvest and Sowing, and it is not the place here to dwell on the strange vicissitudes that have befallen this notion. Two of the master-themes of the new geometry are, however, derived from that of the topos, two complementary "cohomological theories", both conceived in order to provide an approach to Weil's conjectures: the étale (or "l-adic") theme, and the crystalline theme. The former has been concretised in my hands into the l-adic cohomological tool, which now appears as one of the most powerful mathematical tools of the century. As for the crystalline theme, reduced after my departure to a quasi-occulent existence, it was finally exhumed (under the pressure of needs) in June 1981, under the spotlight and under an alias, in circumstances even stranger than those surrounding the topos.

The l-adic cohomological tool was, as expected, the essential tool to establish Weil's conjectures. I demonstrated a good number of them myself, and the last step was accomplished with mastery, three years after my departure, by Pierre Deligne, the most brilliant of my "cohomological" students.

In fact, around 1968, I had developed a stronger and, above all, more 'geometrical' version of Weil's conjectures. These remained 'tainted' (if one may say so!) by an apparently irreducible 'arithmetic' aspect, even though the very spirit of these conjectures is to express and grasp 'the arithmetic' (or 'the discrete') through the mediation of the 'geometric' (or 'the continuous')([1]). In this sense, the version of the conjectures that I had drawn up seems to me to be more "faithful" than Weil's own to "Weil's philosophy" - to that unwritten and rarely spoken philosophy, which has perhaps been the main tacit motivation in the extraordinary rise of geometry over the last four decades([2]). My re-formulation consisted, in essence, in extracting a kind of "quintessence" of what was to remain valid, in the framework of the so-called "abstract" algebraic varieties, of the classical "Hodge theory", valid for the "ordinary" algebraic varieties([3]). I have called this new, entirely geometric version of the famous conjectures "standard conjectures" (for algebraic cycles).

In my mind, this was a new step, after the development of the l-adic cohomological tool, towards these conjectures. But at the same time and above all, it was also one of the possible principles of approach to what still appears to me as the most profound theme I have introduced into mathematics([4]): that of motives (itself born of the "l-adic cohomological theme"). This theme is like the heart or soul, the most hidden part, the best hidden from view, of the schematic theme, which itself is at the heart of the new vision. And the few key-

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[1] (For the mathematician) Weil's conjectures are subject to assumptions of an "arithmetical" nature, in particular because the varieties under consideration must be defined over a finite field. From the point of view of the cohomological formalism, this leads to give a special place to the Frobenius endomorphism associated to such a situation. In my approach, the crucial properties (such as the "generalised index theorem") works for*[concernent]* any algebraic correspondences, and do not make any arithmetic assumption on a previously given base field.

[2] However, after I left in 1970, there was a clear reaction, which resulted in a situation of relative stagnation, which I have mentioned more than once in Harvest and Sowing.

[3] "Ordinary" here means: "defined on the field of complex numbers". Hodge's theory (the so-called "harmonic integral") was the most powerful cohomological theory known in the context of complex algebraic varieties.

[4] This is the most profound theme, at least in the "public" period of my activity as a mathematician, between 1950 and 1969, that is, until I left the mathematical scene. I consider the theme of anabelian algebraic geometry and Galois-Teichmüller theory, developed from 1977 onwards, as being of comparable depth.

phenomena revealed in the standard conjectures(1) can be seen as forming a kind of ultimate quintessence of the motivic theme, as the vital "breath" of this most subtle of all themes, of this "heart within the heart" of the new geometry.

Here is roughly what it is all about. We have seen, for a given prime number p, the importance (especially in view of Weil's conjectures) of knowing how to construct "cohomological theories" for "(algebraic) varieties of characteristic p". Now, the famous "l-adic cohomological tool" provides just such a theory, and even an infinite number of different cohomological theories, namely one associated with any prime number l different from characteristic p. Then clearly there is a "missing theory", which would correspond to the case of an l equal to p. To provide such a theory, I have imagined yet another cohomological theory (which has already been alluded to earlier), called "crystalline cohomology". Moreover, in the important case where p is infinite, there are three other cohomological theories(2) - and there is nothing to prove that we will not be led, sooner or later, to introduce yet more cohomological theories, with similar formal properties. Contrary to what happened in ordinary topology, we are therefore faced with a disconcerting abundance of different cohomological theories. One had the very clear impression that, in a sense that remained rather vague at first, all these theories should "amount to the same thing", that they "gave the same results" (3). In order to express this intuition of "kinship" between different cohomological theories, I have introduced the notion of "motive" associated to an algebraic variety. By this term, I mean to suggest that it is the "common motive" (or "common reason") underlying this multitude of different cohomological invariants associated with the variety, using the multitude of all possible cohomological theories a priori. These different cohomological theories would be like so many different thematic developments, each in its own "tempo", "key" and "mode" ("major" or "minor"), of the same "base motive" (called "motivic cohomology theory"), which would be, at the same time, the most fundamental, or "finest", of all these different thematic "incarnations" (that is to say, of all these possible cohomological theories). Thus, the motive associated with an algebraic variety would constitute the "ultimate" cohomological invariant, "par excellence", from which all the others (associated with the different possible cohomological theories) would be deduced, as many different musical "incarnations", or "realisations". All the essential properties of "the cohomology" of the variety would be "read" (or "heard") already on the corresponding motive, so that the familiar properties and structures on the particularised cohomological invariants (l-adic or

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1 (For the interest of the geometrician reader) These conjectures may need to be rephrased. For more detailed comments, see "Le tour des chantiers" (ReS IV note no. 178, p. 1215-1216) and the footnote on p. 769 in "Conviction et connaissance" (ReS III, note no. 162).

2 (For the interest of the mathematician reader) These theories correspond respectively to Betti cohomology (defined by a transcendental way, using a extension of the base field in the field of complex numbers), Hodge cohomology (defined by Serre) and De Rham cohomology (defined by me), the latter two dating back to the fifties (and Betti's, to the last century).

3 (For the interest of the mathematical reader) For example, if f is an endomorphism of the algebraic variety X , inducing an endomorphism of the cohomology space H^i (X), the "characteristic polynomial" of the latter should have integer coefficients, not depending on the particular cohomological theory chosen (for example l-adic, for l variable). It is the same for general algebraic correspondences, when X is assumed to be proper and smooth. The sad truth (and one which gives an idea of the lamentable state of abandonment of the cohomological theory of algebraic varieties in characteristic p > 0, since I left), is that this thing is still not proved at the moment, even in the particular case where X is a projective and smooth surface and i = 2. In fact, as far as I know, no one after my departure has yet deigned to be interested in this crucial question, typical of those that appear to be subordinate to the standard conjectures. The decree of fashion, is that the only endomorphism worthy of attention is the Frobenius endomorphism (which may have been treated separately by Deligne, using the available methods...).

crystalline, for example), would simply be a faithful reflection of the properties and structures internal to the motive ([1]).

This, expressed in the non-technical language of a musical metaphor, is the quintessence of an idea that is still childishly simple, yet delicate and bold. I developed this idea, in the margins of the foundational tasks I considered more urgent, under the name of 'motive theory' or 'motive philosophy (or 'yoga')', throughout the years 1963-69. It is a theory of fascinating structural richness, much of which is still conjectural([2]).

I spoke on several occasions in Harvest and Sowing about this "yoga of motives", which is particularly dear to my heart. This is not the place to go back over what I have said elsewhere. Suffice it to say that the "standard conjectures" arise most naturally from this yoga of motives. At the same time they provide a principle of approach for one of the possible constructions*[constructions en forme]* of the notion of motive.

These conjectures seemed to me, and still do, to be one of the two most fundamental questions in algebraic geometry. Neither this question, nor the other equally crucial one (the so-called "resolution of singularities") has yet been solved. But while the second of these questions appears, today as it did a hundred years ago, as a prestigious and formidable question, the one I had the honour of bringing up has been classified by the peremptory decrees of fashion (as early as the years following my departure from the mathematical scene, and

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[1] (For the interest of the mathematical reader) Another way of looking at the category of motives over a field k is to visualise it as a kind of "enveloping abelian category" of the category of separated schemes of finite type over k. The motive associated with such a scheme X (or "motivic cohomology of X", which I note H*_{mot}(X)) thus appears as a kind of abelianised "avatar" of X. The crucial thing here is that, just as an algebraic variety X is susceptible to "continuous variation" (its isomorphic class thus depends on continuous "parameters", or "moduli"), the motive associated with X , or more generally, a "variable" motive, is also susceptible to continuous variation. This is an aspect of motivic cohomology, which is in striking contrast to what happens for all classical cohomological invariants, including l-adic invariants, with the sole exception of the Hodge cohomology of complex algebraic varieties.

This gives an idea of how much finer the "motivic cohomology" invariant is, and how much tighter the "arithmetic form" (if I dare to use this expression) of X is, than the traditional purely topological invariants. In my vision of motives, these constitute a kind of very hidden and delicate "cord", linking the algebraic-geometric properties of an algebraic variety, to properties of an "arithmetic" nature embodied by its motive. The latter can be considered as an object of a "geometrical" nature in its very spirit, but where the "arithmetical" properties overlaid on the geometry are, so to speak, "laid bare”.

Thus, the motive appears to me as the deepest "invariant of the form" that we have been able to associate so far to an algebraic variety, apart from its "motivic fundamental group". Both invariants represent for me the "shadows" of a "type of motivic homotopy" which remains to be described (and about which I say a few words in passing in the note "Le tour des chantiers - ou outils et vision" (ReS IV, no. 178, see chantier 5 (Motifs), and in particular page 1214). It is this last object which seems to me to be the most perfect incarnation of the elusive intuition of "arithmetic form" (or "motivic form") of any algebraic variety.

[2] I explained my vision of the motives to anyone who wanted to hear it, throughout these years, without bothering to publish anything about it in black and white (not lacking other tasks in the service of all). This allowed some of my students to plunder more easily later on, under the tenderised eye of all my former friends, who were well aware of the situation. (See following footnote.)

just like the motivic theme itself([1])) as an amiable Grothendieckian joke*[fumisterie]*. But once again I anticipate...

17. Discovering the Mother - or the two sides

To tell the truth, my reflections on Weil's conjectures themselves, with the aim of establishing them, remained sporadic. The panorama that had begun to open up before me and that I was trying to scrutinise and capture far exceeded in breadth and depth the supposed needs of a proof, and even all that these famous conjectures had first been able to suggest. With the appearance of the schematic theme and that of the topos, a new and unsuspected world had suddenly opened up. "The conjectures" occupied a central place in it, certainly, rather like the capital of a vast empire or continent, with countless provinces, but most of which have only the most distant relations with this brilliant and prestigious place. Without ever having to tell myself, I knew that I was now the servant of a great task: to explore this immense and unknown world, to grasp its contours as far as the most distant frontiers; and also to travel in all directions and make an inventory with tenacious and methodical care of the nearest and most accessible provinces, and to draw up maps of scrupulous fidelity and precision, where the smallest hamlet and the smallest cottage would have their place...

It was this last work in particular that absorbed most of my energy - a patient and vast work of foundations that only I could see clearly and, above all, 'feel with my gut'. It took up, by far, the largest part of my time, between 1958 (the year when the schematic and topos themes appeared, one after the other) and 1970 (the year I left the mathematical scene).

I often had to gnaw at the fact that I was held back like this, as if by a tenacious and sticky weight, with these endless tasks which (once I had seen the essentials) were more akin to "stewardship" for me, than to a launch into the unknown. I constantly had to hold back the impulse to move forward - the impulse of the pioneer or the explorer, to discover and explore unknown and unnamed worlds, constantly calling me to know them and to name them. This impulse, and the energy I invested in it (as if by stealth, almost!), were constantly kept to a minimum.

Yet I knew deep down that it was this energy, stolen (so to speak) from the energy I devoted to my 'tasks', that was of the rarest and most delicate essence - that the 'creation' in my work as a mathematician belonged <u>there</u> above all: in this intense attention to apprehend, in the obscure, shapeless and moist folds of a warm and inexhaustible nourishing womb*[matrice]*, the first traces of form and outline of what had not yet been born and which seemed to be calling me, to take shape and become incarnate and be born... In the work of discovery, this intense attention, this ardent solicitude, is an essential force, like the warmth of the sun for the obscure gestation of seeds buried in the nourishing earth, and for their humble and miraculous blossoming into the light of day.

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[1] In fact, this theme was exhumed in 1982 (one year after the crystalline theme), under its original name this time (and in a narrowed form, in the sole case of a base field of zero characteristic), without the name of the worker being pronounced. This is one example among many others of a notion or theme buried as Grothendieckian phantasmagoria in the aftermath of my departure, only to be exhumed one after the other by some of my students over the next ten or fifteen years, with modest pride and (need I say it again) without mention of the worker...

In my work as a mathematician, I see these two forces or impulses at work, equally deep, of (it seems to me) different natures. To evoke both, I have used the image of the builder, and that of the pioneer or explorer. Placed side by side, both suddenly strike me as very 'yang', very 'masculine', even 'macho'! They have the haughty resonance of myth, or that of "great occasions". Surely they are inspired by the vestiges, in me, of my former "heroic" vision of creative work, the super-yang vision. As they stand, they give a strongly tinted, not to say fixed, "at attention" vision of a much more fluid, more humble, more "simple" reality - of a living reality.

In this male impulse of the 'builder', which seems to push me constantly towards new building sites, I discern, however, at the same time, that of the homebound: of the one deeply attached to 'the' house. Before anything else, it is 'his' house, the house of his 'close ones*[proches]*' - the place of an intimate living entity of which he feels a part. Only then, and as the circle of what is felt to be 'close' widens, is it also a 'home for all'. And in this impulse to "make homes" (as one would "make love"...) there is also and above all a tenderness. There is the impulse of contact with these materials that one shapes one by one, with loving care, and which one only really knows through this loving contact. And, once the walls are up and the beams and roof are in place, there is the deep satisfaction of installing one room after another, and of seeing implementing, little by little, among these rooms, chambers and storerooms, the harmonious order of the living house - beautiful, welcoming, good for living in. For the house, first and foremost and secretly in each of us, is also the mother - that which surrounds and shelters us, both refuge and comfort; and perhaps (even more profoundly, and even as we would be building it from scratch) it is also that from which we ourselves came, that which sheltered and nourished us, in those forever-forgotten times before we were born... It is also the Bosom*[Giron]*.

And the image that spontaneously appeared earlier, in order to go beyond the prestigious appellation of "pioneer", and to identify the more hidden reality that it covered, was also stripped of any "heroic" accent. Again, it was the archetypal image of the maternal that appeared - that of the nurturing 'womb' and its shapeless, obscure labours...

These two impulses which appeared to me as "different in nature" are in the end closer than I would have thought. Both are in the nature of a "contact impulse", bringing us to the meeting of "the Mother": of She who embodies both what is close, "known", and what is "unknown". To surrender to one or the other impulse is to "find the Mother". It is to renew contact with both the near, the "more or less known", and the "distant", with what is "unknown" but at the same time sensed, on the verge of being known.

The difference here is one of tone, of dosage, not of nature. When I 'build houses', it is the 'known' that dominates, and when I 'explore', it is the unknown. These two "modes" of discovery, or to put it better, these two aspects of the same process or the same work, are indissolubly linked. They are both essential and complementary to one another. In my mathematical work, I discern a constant back-and-forth movement between these two modes of approach, or rather, between the moments (or periods) when one predominates, and those when the other predominates([1]). But it is also clear that in each moment, both modes are present. When I am building, arranging, or clearing, cleaning, ordering, it is the "yang" or "masculine" "mode" or "side" of the work that sets the tone. When I explore and grope the elusive, the formless, the nameless, I am the "ying", or "feminine" side of my being.

It is not a question of me wanting to minimise or deny either side of my nature, both are essential - the 'masculine' that builds and generates, and the 'feminine' that conceives, and shelters the slow and obscure gestations. I 'am' both - 'yang' and 'yin', 'male' and 'female'. But I also know that the most delicate, most loose*[déliée]* essence in the creative processes lies on the side of the "yin", "feminine" - the humble, obscure, and often poor-looking side.

It is this side of work that I believe has always held the most powerful fascination for me. The prevailing consensus, however, encouraged me to invest most of my energy in the other side, in that which is embodied and affirmed in tangible, not to say finished and completed "products" - products with clear-cut contours, attesting to their reality with the evidence of cut stones...

I can see, with hindsight, how this consensus weighed on me, and also how I 'bore the weight' - in a flexible way! The 'design' or 'exploration' part of my work was kept to a minimum until the moment of my departure, it is true. And yet, in this retrospective glance at my work as a mathematician, it is strikingly obvious that the essence and the power of this work is indeed this side which is nowadays neglected, when it is not the object of derision or condescending disdain: that of the 'ideas', or even that of the 'dream', yet never that of the 'results'. Trying in these pages to identify the most essential contribution I have made to the mathematics of my time, by looking at a forest rather than at trees - I have seen, not a honour list of "great theorems", but a lively range of fruitful ideas([2]), all contributing to a single vast vision.

18. The child and the mother

When this "foreword" started to turn into a promenade through my work as a mathematician, with my little talk about "inheritors" (good-natured) and "builders" (incorrigible), a name also started to appear for this failed

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[1] What I say here about mathematical work is also true of the work of "meditation" (which will be discussed throughout Harvest and Sowing). There is little doubt in my mind that this is something that appears in all work of discovery, including that of the artist (writer or poet, say). The two "sides" I am describing here can be seen equally as one of expression and its "technical" demands, and the other of reception (of perceptions and impressions of all kinds), which become inspiration through intense attention. Both are present at all times during the work, and there is a constant back-and-forth movement between the "times" when one predominates and those when the other predominates.

[2] It is not that there is a lack of what might be called "great theorems" in my work, including theorems that solve questions posed by others, which no one before me had been able to solve. (I will review some of these in the footnote (***) page 554, of the note "The rising sea..." (ReS III, no. 122).) But, as I have already underlined at the beginning of this "promenade" (in the step "Points of view and vision", no. 6), these theorems take for me all their meaning only by the nourishing context of a great theme, initiated by one of these "fertilised ideas". Their proof then follows, as if from the source and without effort, from the very nature, from the "depth" of the theme that carries them - as the waves of the river seem to be born smoothly from the very depth of its waters, without rupture and without effort. I will express myself in a very similar vein, but with other images, in the note already quoted "The rising sea...".

foreword: it would be "The child and the builder". In the course of the following days it became increasingly clear that 'the child' and 'the builder' were one and the same character. So the name became, more simply, "The Child Builder". A name, my goodness, that was not lacking in allure, and all to my liking!

But then it becomes clear that this haughty "builder", or (more modestly) the child-who-played-at-making-houses, was only one face of the famous child-who-played, who had <u>two</u>. There is also the child-who-likes-to-explore-things, to go snooping[fouiner] and burrowing[enfouir] in the sands or in the muddy and nameless sludge, the most impossible and the most absurd places... To give the impression (if only to myself...), I began by introducing him under the flamboyant name of "pioneer", followed by the more down-to-earth but still prestigious "explorer". It made you wonder which of the 'builder' and the 'pioneer-explorer' was the more male, the more attractive of the two! Heads or tails?

And then, on closer inspection, our intrepid "pioneer" turns out to be a <u>girl</u> (whom I had liked to dress as a boy) - a sister of the ponds, of the rain, of the mists and of the night, silent and almost invisible by dint of fading into the shadows - the one who is always forgotten (when we don't even pretend to laugh at her...). And I too found a way, for days and days, to forget her - to forget her doubly, I might say: I had only wanted to see the boy at first (the one who plays at making houses...) - and even when I couldn't help but, by force, to see <u>the other one</u> as well, I still saw a boy, her too as a boy...

As for the beautiful name for my promenade, it doesn't hold up at all. It's an all-in-yang name, all "macho", a name-that-boxes. To make it not lopsided, you'd have to put <u>the other one</u> in there too. But, strangely enough, <u>"the other one" doesn't really have a name</u>. The only one that fits*[colle]* is "explorer", but it's still a boy's name, nothing we can do. The language here is a bitch, it traps us without us even realising it, obviously in fuse*[mêche]* with age-old prejudices.

Perhaps we could get away with "The child-who-builds and the child-who-explores". Leaving unsaid that one is "boy" and the other is "girl", and that it is one and the same boy-girl child who, in building, explores, and in exploring, builds... But yesterday, in addition to the double yin-yang aspect of what contemplates and explores, and what names and builds, another aspect of things had appeared.

The Universe, the World, even the Cosmos, are basically foreign and very distant things. They do not really concern us. It is not towards them that the impulse for knowing carries us deep within ourselves. What attracts us, is their tangible and immediate <u>Incarnation</u>, the closest, the most "carnal", charged with deep resonances and rich in mystery - the One who merges with the origins of our being of flesh, as with those of our species - and the One who has always waited for us, silent and ready to welcome us, "at the other end of the path". It is from <u>Her</u>, the Mother, from Her who gave birth to us as She gave birth to the World, that the impulse arises and that the paths of desire soar - and it is to <u>Her</u> encounter that they carry us, to <u>Her</u> that they soar, to return unceasingly and to collapse*[s'abîmer]* into Her.

Thus, at the bend in the road of an unexpected "promenade", I unexpectedly find a parable that was familiar to me, and that I had somewhat forgotten - the parable of <u>the child and the Mother</u>. It can be seen as a parable for "<u>Life, in search of itself</u>". Or, on the more humble level of individual existence, a parable for "<u>Being, in search of things</u>".

It is a parable, and it is also the expression of an ancestral experience, deeply implanted in the psyche - the most powerful of the original symbols that nourish the deep creative layers. I believe I recognise in it, expressed in the immemorial language of archetypal images, the very breath of creative power in man, animating his flesh and spirit, in its humblest and most ephemeral, as well as its most brilliant and lasting manifestations.

This "breath", like the carnal image that embodies it, is the most humble thing in the world. It is also the most fragile thing, the most ignored by all and the most despised...

And the story of the vicissitudes of this soufflé in the course of your existence is none other than <u>your</u> adventure, the "adventure of knowledge" in <u>your</u> life. The wordless parable that expresses it is that of the child and the Mother.

You are the child, born of the Mother, sheltered in Her, nourished by Her power. And the child soars from the Mother, the All-knowing*[Toute-proche]*, the Well-known - to meet the Mother, the Unlimited, forever Unknown and full of mystery...

End of the "Promenade through an œuvre”

Epilogue: <u>the invisible Circles</u>

19. Death is my cradle (or three kids for a moribund)

Until the appearance of the topos point of view, towards the end of the fifties, the evolution of the notion of space appears to me as an essentially "continuous" evolution. It seems to have continued smoothly and without jumps, starting from the Euclidean theorisation of the space that surrounds us, from the geometry bequeathed by the Greeks, focusing on the study of certain "figures" (lines, planes, circles, triangles, etc.) living in this space. Certainly, profound changes have taken place in the way the mathematician or the "natural philosophers" conceived the notion of the “space”([1]). But these changes all seem to me to be in the nature of an essential "continuity" - they never presented the mathematician, attached (like everyone else) to familiar mental images, with a sudden <u>change of scenery[dépaysement]</u>. They were like the changes, possibly profound but incremental, that take place over the years in a being that we knew as a child, and whose evolution we followed from his first steps to his adulthood and to his full maturity. Changes that are imperceptible in some long periods of deep calm, and tumultuous perhaps in others. But even in the most intense periods of growth or maturation, and even

[1] My initial intention in writing the Epilogue was to include a very rough sketch of some of these "profound changes", and to show the "essential continuity" that I see in them. I have given up on this, so as not to overly lengthen this Promenade, which is already much longer than expected! I am thinking of coming back to it in the Historical Comments planned for volume 4 of the "Reflections", this time for a mathematician reader (which would totally change the way of exposition).

if we had lost sight of him for months or even years, at no time could there be the slightest doubt, the slightest hesitation: it was still him, a well-known and familiar being, whom we could always recognised, even if with changed features.

I think I can say that by the middle of this century, this familiar being had already grown very old - like a man who had finally become exhausted and worn out, overwhelmed by an influx of new tasks for which he was in no way prepared. Perhaps he had already died a good death, without anyone bothering to take note of it and acknowledge it. "Everyone" was still pretending to be busy in the house of a living person, so that it was almost as if he were still alive and well enough.

Yet now, observe the unfortunate effect for the regular guests of the house, when in place of the venerable old man, frozen, stiff and upright in his armchair, there suddenly appears a vigorous boy, no taller than three apples, who claims in passing, in a serious manner and as if it is something self-evident, that Mr. Space (and you can even drop the "Mr." now, at your leisure...) is him! If only he looked like he had the family traits, a natural child perhaps, something like that... but not at all! From the looks of it, nothing reminds us of the old Father Space we had known so well (or thought we did...), and of whom we were quite sure that, at any rate (and nothing can be true if this is not...), he was eternal...

This is **it**, the famous "mutation of the notion of space". This is the thing that I must have 'seen', as something obvious, from the beginning of the sixties at least, without ever having had the opportunity to formulate it to myself before this very moment when I am writing these lines. And I suddenly see with a new clarity, by the mere virtue of this pictorial evocation and the cloud of associations it immediately arouses: the traditional notions of "space", just like the closely related notion of "variety" (of all kinds, and in particular that of "algebraic variety"), had become so old by the time I came around, that it was as if they were already dead... ([1]). And I could say that it is with the appearances, one after the other, of the point of view of schemes (and its progeny([2]), plus ten thousand pages of foundations), and then of that of the topos, that a situation of crisis-which-doesn't-say-its-name was finally unravelled.

In the image above, it's not just one kid from outside that should be mentioned, as the product of a sudden mutation, but two. Two kids, moreover, who have an unmistakable "family resemblance" to each other, even if

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[1] This assertion (which will seem peremptory to some) should be taken with a 'grain of salt'. It is not more nor less valid than the statement (which I will take up again below) that the "Newtonian model" of mechanics (terrestrial or celestial) was "moribund" at the beginning of this century, when Einstein came to the rescue. It is a fact that even today, in most "common" situations in physics, the Newtonian model is perfectly adequate, and it would be crazy (given the margin of error allowed in the measurements made) to go looking for relativistic models. Similarly, in many mathematical situations, the old familiar notions of "space" and "variety" remain perfectly adequate, without going in search of nilpotent elements, topos or "moderate structures". But in both cases, for an increasing number of contexts involved in frontier research, the old conceptual frameworks have become inadequate to express even the most "common" situations.

[2] (For mathematician) In this " progeny ", I count in particular the formal schemes, "multiplicities" of all kinds (and in particular, schematic or formal multiplicities), and finally the so-called "rigid-analytic" spaces (introduced by Tate, following a " project manager*[maître d'œuvre]* " provided by me, inspired by the new notion of topos, as well as by that of formal schemes). This list is by no means exhaustive.

they hardly resemble the late old man. And if you look closely, you could say that the toddler Schemes would be like a "link of kinship" between the late Father Space (aka Varieties-of-all-types) and the toddler Topos([1]).

20. A look at the neighbours across the street

The situation seems to me to be very similar to that which arose at the beginning of this century with the appearance of Einstein's theory of relativity. There was an even more glaring conceptual dead end, taking the form of a sudden contradiction, which seemed irresolvable. Not surprisingly, the new idea that brought order to the chaos was an idea of childlike simplicity. The remarkable thing (yet conforming to the most recurring scenario...) is that among all the brilliant, eminent, prestigious people who were suddenly under great pressure*[sur les dents]*, trying to "save the furniture", no one thought of this idea. It took an unknown young man, fresh from the benches of the student lecture halls (perhaps a little embarrassed by his own audacity...) to explain to his illustrious elders what had to be done to 'save the phenomena': space and time had to be separated more([2])*[as original - Trans.]*! Technically, everything was in place for this idea to emerge and be accepted. And it is honourable for Einstein's elders to indeed be able to welcome the new idea without too much grumbling*[morigéner]*. This is a sign that it was still a great age...

From the mathematical point of view, Einstein's new idea was trivial. From the point of view of our conception of physical space, on the other hand, it was a profound mutation, and a sudden 'change of scenery'. It was the first mutation of its kind since the mathematical model of physical space developed by Euclid 2400 years ago, and taken up unchanged for the needs of mechanics by all physicists and astronomers since antiquity (including Newton), to describe terrestrial and stellar mechanical phenomena.

This initial idea of Einstein's was later greatly expanded into a more subtle, richer and more flexible mathematical model, using the rich arsenal of mathematical notions already in existence([3]). With the "theory of generalised relativity", this idea was expanded into a vast vision of the physical world, embracing in a single view the subatomic world of the infinitely small, the solar system, the Milky Way and distant galaxies, and the course of electromagnetic waves in a space-time curved at every point by the matter contained therein([4]). This is

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[1] In addition to these two toddlers, we should add a third, younger one, who appeared in less favourable times: this is the little kid called Tame Space. As I have pointed out elsewhere, he did not have the right to a birth certificate, and it is in total illegality that I have nevertheless included him among the twelve 'master themes' that I had the honour of introducing into mathematics.

[2] It is a bit short, of course, as a description of Einstein's idea. At the technical level, it was necessary to reveal what structure should be put on the new space-time (though this was already "in the air", with Maxwell's theory and Lorenz's ideas). The essential step here was not technical, but "philosophical": to realise that the notion of simultaneity for distant events had no experimental reality. This is the "childish realisation", the "but the Emperor is naked!", which led to the crossing over of this famous "imperious and invisible circle which limits a Universe"...

[3] It is mainly about the notion of "Riemannian variety", and the tensor calculus on such a variety.

[4] One of the most striking features that distinguishes this model from the Euclidean (or Newtonian) model of space and time, and also from Einstein's very first model ("special relativity"), is that the global topological form of space-time remains undetermined, instead of being imperatively prescribed by the very nature of the model. The question of what this global form is, it seems to me (as a mathematician), one of the most fascinating questions in cosmology.

the second and last time in the history of cosmology and physics (following Newton's first great synthesis three centuries ago) that a vast unifying vision, in the language of a mathematical model, of all physical phenomena observed in the Universe.

This Einsteinian vision of the physical Universe in fact has, in turn, been overwhelmed by new developments. The "all physical phenomena observed" that we are trying to account for has had plenty of time to expand since the beginning of the century! A multitude of physical theories have appeared, each one trying to account, with varying degrees of success, for a limited number of facts, in the immense muddle of all the "observed facts".

And we are still waiting for the audacious kid, who will find by playing the new key (if there is one...) the dreamed "model-cake", which will be willing to “work*[marcher]*" to save all the phenomena at once... ([1])

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[1] Such a hypothetical theory, which would manage to "unify" and reconcile the multitude of partial theories that have been mentioned, has been called a " unified theory ". I have the feeling that the fundamental reflection that awaits to be undertaken will have to be placed on two different levels.

1) A reflection of a "philosophical" nature, on the very notion of a "mathematical model" for a portion of reality. Since the success of the Newtonian theory, it has become a tacit axiom of the physicist that there **exists** a mathematical model (or even a unique model, or "**the**" model) to express physical reality in a perfect way, without "detachment" or blunder. This consensus, which has been the law for more than two centuries, is like a kind of fossil vestige of the vivid vision of a Pythagorean "All is number". Perhaps this is the new "invisible circle", which has replaced the old metaphysical circles to limit the physicist's Universe (while the race of the " natural philosophers " seems to be extinct for good, supplanted high-handedly by that of the computers...). If one is willing to stop for a moment, it is clear that the validity of this consensus is not at all obvious. There are even very serious philosophical reasons for questioning it a priori, or at least for placing very strict limits on its validity. This would be the moment or never to subject this axiom to close criticism, and perhaps even to 'demonstrate', beyond any possible doubt, that it is **un**founded: that there exists **no** single rigorous mathematical model that accounts for all of the so-called 'physical' phenomena recorded to date.

Once the very notion of a "mathematical model" has been satisfactorily defined, and that of the "validity" of such a model (within the limits of such "margins of error" admitted in the measurements made), the question of a "unified theory" or at least that of an "optimum model" (in a sense yet to be specified) will then finally be clearly posed. At the same time, we will no doubt have a clearer idea of the degree of arbitrariness that is attached (by necessity, perhaps) to the choice of such a model.

2) It seems to me that it is only <u>after</u> such a reflection can the "technical" question of finding an explicit model, more satisfactory than its predecessors, take on its full meaning. This would be the moment, perhaps, to free ourselves from a second tacit axiom of the physicist, dating back to antiquity, and deeply anchored in our very mode of perception of space: it is that of the <u>continuous nature</u> of space and time (or of space-time), therefore of the “place" where "physical phenomena" take place.

Fifteen or twenty years ago, while leafing through the small volume of Riemann's complete works, I was struck by a remark he made 'in passing'. In it he observed that the ultimate structure of space may well be 'discrete', and that the 'continuous' representations we make of it may be a simplification (and possibly, over time, an over-simplification) of a more complex reality; that for the human mind, 'the continuous' was easier to grasp than 'the discontinuous', and that it therefore serves as an 'approximation' to grasp the discontinuous. This is a surprisingly penetrating remark from a mathematician, at a time when the Euclidean model of physical space had never been challenged; yet in a strictly logical sense, it is the discontinuous which, traditionally, has served as the technical approach to the continuous.

Developments in mathematics in recent decades have shown a much closer symbiosis between continuous and discontinuous structures than was imagined in the first half of this century. Still, to find a "satisfactory" model (or, if need be, a set of such models, "connecting" each other as satisfactorily as possible...), be it " continuous ", " discrete " or of a "mixed" nature - such a work will surely require a great conceptual imagination, and a consummate flair for apprehending and uncovering mathematical structures of a new type. This kind of imagination or "flair" seems to me to be rare, not only among physicists (where Einstein and Schrödinger seem to have been among the rare exceptions), but even among mathematicians (and here I can speak with full knowledge of the matter).

To sum up, I predict that the expected renewal (if it is to come at all...) will come from someone who is a mathematician in the soul, and well informed about the great problems of physics, rather than from a physicist. But above all, it will require a man with the "philosophical openness" to grasp the crux of the problem. This is not a technical problem at all, The comparison between my contribution to the mathematics of my time and Einstein's contribution to physics came to mind for two reasons: both works are accomplished through a mutation of our conception of "space" (in the mathematical sense in one case, and physical sense in the other); and both take the form of a unifying vision, embracing a vast multitude of phenomena and situations which until then had appeared to be separate from one another. I see an obvious kinship of spirit between his work([1]) and mine.

This kinship does not seem to me to be contradicted by an obvious difference in "substance". As I have already suggested, the Einsteinian mutation concerns the notion of physical space, whereas Einstein drew on the arsenal of mathematical notions already known, without ever needing to enlarge or even overturn it. His contribution consisted in identifying, among the mathematical structures known at the time, those that were best suited to serve as 'models' of the world of physical phenomena, in place of the moribund([2]) model bequeathed by his predecessors. In this sense, his work was indeed that of a **physicist**, and beyond that, that of a "natural philosopher", in the sense that Newton and his contemporaries understood it. This "philosophical" dimension is absent from my mathematical work, where I have never been led to ask myself questions about the possible relations between the "ideal" conceptual constructions, taking place in the Universe of mathematical things, and the phenomena which take place in the physical Universe (or even, the experienced events taking place in the psyche). My work has been that of a mathematician, deliberately turning away from the question of 'applications' (to other sciences), or the 'motivations' and psychic roots of my work. A mathematician, on top, who is driven by his very particular genius to constantly expand the arsenal of notions at the very basis of his art. This is how I was led, without even realising it and as if by playing, to overturn the most fundamental notion of all for the geometers: that of space (and that of "variety"), that is to say our conception of the very "**place**" where geometric beings live.

The new notion of space (as a kind of "generalised space", but where the points that are supposed to form the "space" have more or less disappeared) bears no resemblance in substance to the notion brought by Einstein to physics (which, for the mathematicians, is not at all puzzling). On the other hand, the comparison is with the quantum mechanics discovered by Schrödinger([3]). In this new mechanics, the traditional "material point" disappears, being replaced by a sort of "probabilistic cloud", of varying density from one region of the ambient space to another, depending on the "probability" that the point is located in that region. In this new perspective, one can feel a deeper "mutation" in our way of conceiving mechanical phenomena than in the one embodied by Einstein's model - a mutation that does not consist in simply replacing a mathematical model that is a bit narrow around the edges by a similar one that is wider or better fitted. This time, the new model bears so little resemblance to the good old traditional models that even the mathematician who is a great specialist in mechanics must have felt suddenly out of place, even lost (or outraged...). To go from Newton's mechanics to

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[1] I make no claim to be familiar with Einstein's work. In fact, I have not read any of his work, and know his ideas only by hearsay and very roughly. Yet I have the impression of discerning "the forest", even though I have never had to make the effort to carefully examine any of its trees...

[2] For comments on the qualifier "moribund", see a previous footnote (note (*) page 55 *[note (2)"This assertion..."- Trans.]*).

[3] I understand (from echoes that have come back to me from various sources) that it is generally considered that there have been three "revolutions" or great upheavals in physics this century: Einstein's theory, the discovery of radioactivity by the Curies, and the introduction of quantum mechanics by Schrödinger.

Einstein's must be, for the mathematicians, a bit like going from the good old Provençal dialect to the latest Parisian slang. On the other hand, passing to quantum mechanics, I imagine, is like passing from French to Chinese.

And these "probabilistic clouds", replacing the reassuring material particles of yesteryear, strangely remind me of the elusive "open neighbourhoods" that inhabit the topos, like evanescent phantoms, to surround imaginary "points", to which a recalcitrant imagination continues to cling against all odds...

21. "The One" - or the gift of solitude

This brief excursion to the 'neighbours across the street', the physicists, may serve as a point of reference for a reader who (like most people) knows nothing of the world of mathematicians, but who has surely heard of Einstein and his famous 'fourth dimension', or even of quantum mechanics. After all, even if the inventors didn't expect that their discoveries would result in Hiroshima, and later in both military and (supposedly) 'peaceful' atomic escalations, the fact is that discovery in physics has a tangible and almost immediate impact on the human world in general. The impact of mathematical discoveries, especially in the so-called "pure" mathematics (that is to say, without motivation for "applications") is less direct, and certainly more delicate to determine. I am not aware, for example, that my contributions to mathematics have been 'used' for anything, say to build any kind of machine. I don't deserve any credit for that, that's for sure, but it does reassure me. As soon as there are applications, you can be sure that it's the military (and after them, the police) who are the first to get hold of them - and neither is it so much better when it comes to the industry (even the so-called "peaceful" one)...

For my own sake, of course, or for that of a mathematician reader, it would be better to try to situate my work by means of "reference points" in the history of mathematics itself, rather than to look for analogies elsewhere. I have been thinking about this for the past few days, within the limits of my rather vague knowledge of the history in question([1]). In the course of the "Promenade", I had already had the occasion to evoke a "lineage" of mathematicians, of a temperament in which I recognise myself: Galois, Riemann, Hilbert. If I were better acquainted with the history of my art, there is a chance that I would find it possible to extend this lineage further into the past, or perhaps to insert a few other names that I know little more than hearsay. The thing that struck me is that I do not recall knowing, even by allusion from friends or colleagues better versed in history than I, of any mathematician besides myself who contributed a multiplicity of innovative ideas, not more or less disjoint from each other, but as parts of a vast unifying vision (as was the case for Newton and for Einstein in physics and cosmology, and for Darwin and for Pasteur in biology). I am aware of only two 'moments' in the history of mathematics when a broad new vision was born. One of these moments is the birth of mathematics as a science in the sense we understand it today, 2500 years ago in ancient Greece. The other is, above all, the birth of infinitesimal and integral calculus, in the seventeenth century, a time marked by the names of Newton, Leibnitz,

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[1] Ever since I was a kid, I've never been too keen on history (or geography for that matter). (In the fifth part of Harvest and Sowing (only partially written), I have the opportunity "in passing" to detect what seems to me the deepest reason for this partial "block" against history - a block that is being resorbed, I believe, in recent years). The mathematical education received by my elders, in the "Bourbachian circle", did not help matters - occasional historical references were very rare.

Descartes and others. As far as I know, the vision born at either moment was not the work of a single person, but the collective work of an era.

Of course, between the time of Pythagoras and Euclid and the beginning of the seventeenth century, mathematics had had time to change its face, and likewise between the time of the "Calculus of infinitesimals" created by the mathematicians of the seventeenth century and the milieu of the nineteenth century*[le milieu du présent dix-neuvième]*. But as far as I know, the profound changes that took place during these two periods, one of more than two thousand years and the other of three centuries, were never concretised or condensed into a new vision expressed in a given work([1]), in a way similar to what happened in physics and cosmology, with the great syntheses of Newton and then Einstein, at two crucial moments in their history.

It would seem that as a servant of a vast unifying vision born in me, I am "one of a kind" in the history of mathematics from the beginning to the present. I am sorry if I seem to be trying to singularise myself more than I am allowed to! To my own relief, however, I think I can discern a sort of potential (and providential!) brother. I have already had the opportunity to mention him, as the first in the line of my 'brothers of temperament': it is Evariste Galois. In his short and dazzling life([2]), I believe I can discern the beginnings of a great vision - that of the 'marriage of number and size*[grandeur]*', in a new geometrical vision. I have written elsewhere in Récoltes et Semailles([3]) about how, two years ago, a sudden intuition appeared in me: that in the mathematical work which at that time exerted the most powerful fascination on me, I was in the process of 'taking over the legacy of Galois'. This intuition, rarely mentioned since, has nevertheless had time to mature in silence. The retrospective reflection on my work that I have been pursuing for the past three weeks will surely have

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[1] Hours after writing these lines, it struck me that I had not thought here of the vast synthesis of contemporary mathematics that N. Bourbaki's (collective) treatise endeavours to present. (There will be much more about the Bourbaki group in the first part of Harvest and Sowing.) This is due, it seems to me, to two reasons.

On the one hand, this synthesis is limited to a sort of "putting to order" a vast set of ideas and results already known, without bringing in any new idea of its own. If there is a new idea, it would be that of a precise mathematical definition of the notion of "structure", which has proved to be a precious guiding thread throughout the treatise. But this idea seems to me to be more like that of an intelligent and imaginative lexicographer, than an element of renewal of a language, giving a renewed apprehension of reality (here, of mathematical things).

On the other hand, from the 1950s onwards, the idea of structure was surpassed by new developments, with the sudden influx of 'categorical' methods in some of the most dynamical parts of mathematics, such as topology or algebraic geometry. (Thus, the notion of 'topos' refuses to fit into the 'Bourbachian bag' of structures, which is decidedly narrow around the edges!) In deciding, with full knowledge of the facts, not to enter this "galley", Bourbaki thereby renounced his initial ambition, which was to provide the foundations and the basic language for the whole of contemporary mathematics.

He did, however, set a language and, at the same time, a certain style of writing and approach to mathematics. This style was originally a (very partial) reflection of a certain spirit, a living and direct inheritance from Hilbert. In the course of the fifties and sixties, this style came to prevail - for better and (mostly) for worse. In the last twenty years or so, it has become a rigid 'canon' of purely facade 'rigour', whose spirit that once animated it seems to have disappeared wit no return.

[2] Evariste Galois (1811-1832) died in a duel at the age of twenty-one. There are, I believe, several biographies of him. As a young man, I read a fictionalised biography written by the physicist Infeld, which struck me very much at the time.

[3] See “The legacy of Galois" (ReS I, section 7).

contributed to this. The most direct filiation that I believe I now recognise with a mathematician of the past is that which links me to Evariste Galois. Rightly or wrongly, it seems to me that this vision that I developed during fifteen years of my life, and which has continued to mature in me and to be enriched during the sixteen years that have passed since I left the mathematical scene - this vision is also the one that Galois would have been able to develop([1]), if he had been around in my place, and without an early death brutally cutting short a magnificent momentum.

There is another reason, surely, that contributes to my feeling of an 'essential kinship' - a kinship that cannot be reduced to mere 'mathematical temperament', nor to the outstanding aspects of a work. Between his life and mine, I also feel a kinship of destinies. It is true that Galois died stupidly, at the age of twenty-one, whereas I am about to turn sixty, and am determined to live a long life. However, this does not prevent Evariste Galois, during his life, just like me a century and a half later, from remaining a "marginal" figure in the official mathematical world. In the case of Galois, it might seem to a superficial observation that this marginality was 'accidental', that he had simply not yet had time to 'make his mark' with his innovative ideas and his work. In my case, my marginality, during the first three years of my life as a mathematician, was due to my ignorance (deliberate perhaps...) of the very existence of a world of mathematicians, with which I would have to confront myself; and since my departure from the mathematical scene, sixteen years ago, it is the consequence of a deliberate choice. It is this choice, surely, that has provoked in retaliation a 'unfailing collective will' to erase from mathematics all trace of my name, and with it the vision of which I had made myself the servant.

But beyond these accidental differences, I believe I can discern a common cause to this "marginality", which I feel is essential. I do not see this cause in historical circumstances, nor in particularities of "temperament" or "character" (which are undoubtedly as different from him to me as they can be from one person to another), and certainly even less at the level of "gifts" (obviously prodigious in Galois, and comparatively modest in me). If there is indeed an "essential kinship", I see it at a much more humble and elementary level.

I have felt such a kinship on a few rare occasions in my life.It is also through it that I feel "close" to yet another mathematician, and who was my elder: Claude Chevalley([2]). The link I mean is that of a certain 'naivety', or 'innocence', which I have had occasion to speak of. It is expressed by a propensity (often unappreciated by those around us) to look at things through one's own eyes, rather than through patented glasses, graciously offered by some larger or smaller human group, invested with authority for one reason or another.

This "propensity", or inner attitude, is not the privilege of maturity, but rather that of childhood. It is a gift received at birth, along with life - a humble and formidable gift. It is a gift that is often buried deep, and which some people have managed to preserve to some extent, or perhaps to recover...

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[1] I am convinced that a Galois would have gone much further than I did. On the one hand because of his quite exceptional gifts (which I did not receive). On the other hand, because it is probable that he would not have allowed most of his energy to be distracted, as I did, by endless tasks of meticulously putting into shape, as we go along, what is already more or less acquired...

[2] I mentioned Claude Chevalley here and there in Harvest and Sowing, particularly in the section 'Encounter with Claude Chevalley, or: liberty and good intensions' (ReS I section 11), and in the note 'A farewell to Claude Chevalley' (ReS III, note n° 100).

One may also call it the gift of solitude.

A Letter

May 1985

1. The thousand-page letter 「Omitted」

2. Birth of Harvest and Sowing (a retrospective flashback)

In this pre-letter, I would now like to tell you in a few pages (if possible) what Harvest and Sowing is about - to tell you in a more detailed way than only what the subtitle says: "Reflections and testimony on a mathematician's past" (my past, you must have guessed already...). There are many things in Harvest and Sowing, and different people will no doubt see many different things in it: a <u>voyage</u> to the discovery of a past; a <u>meditation</u> on existence; a <u>picture of the mores</u> of a milieu and an era (or the picture of the insidious and implacable shift from one era to another...); an <u>investigation</u> (almost police-like at times, and at others bordering on the cloak-and-sword novel in the underworld of the mathematical megapolis...); a vast <u>mathematical rambling</u> (which will sow many a seed...); a practical treatise on applied psychoanalysis (or, if you like, a book of "<u>psychoanalysis-fiction</u>"); a panegyric of <u>self-knowing</u>; "My <u>confessions</u>"; an intimate <u>journal</u>; a psychology of <u>discovery and creation</u>; an <u>indictment</u> (merciless, as it should be...), or even a <u>settling of accounts</u> in "the beautiful mathematical world" (and without giving any gifts*[sans faire de cadeaux]*...). What is certain is that at no time was I bored in writing it, while I learned and saw all kinds of things. If your important tasks allow you the leisure to read it, I would be surprised if you were bored while reading my work. Unless you force yourself, who knows...

「Omitted」

3. The death of the boss - derelict building sites

「Omitted」

In these fifteen years of intense mathematical work, a vast <u>unifying vision</u> has hatched, matured and grown within me, embodied in a few very simple <u>key ideas</u>*[idées-force]*. The vision was that of an "arithmetic geometry", a synthesis of topology, geometry (algebraic and analytic), and arithmetic, of which I found a first embryo in the conjectures of Weil. It is this that has been my main source of inspiration in these years, which for me are, above all, those in which I have developed the main ideas of this new geometry, and in which I have shaped some of its main tools. This vision and these key ideas have become second nature to me. (And after having ceased all contact with them for nearly fifteen years, I notice today that this "second nature" is still alive in me!) They were for me so simple, and so obvious, that it went without saying that "the whole world" had gradually assimilated them and made them their own, alongside with me. It is only recently, in these last few months, that I have realised that neither the vision, nor these few "key ideas" which had been my constant guide, are written in full in any published text, except at most between the lines. And above all, that this vision that I thought I was communicating, and these key ideas that carry it, remain today, twenty years after having reached full maturity, ignored by all. It is I, the worker, and servant of these things that I had the privilege of discovering, who am also the only one in whom they are still alive.

This or that tool, which I had fashioned, is used here and there to 'crack' a problem that is deemed difficult, like breaking into a safe. The tool is apparently solid. However, I know it has another 'strength' than that of a pair of monseigneur pliers. It is part of a Whole, as a limb is part of the body - a Whole from which it comes, which gives it its meaning and from which it draws strength and life. You can use a bone (if it's big) to fracture a skull, that's true. But that is not its real function, its raison d'être. And I see these scattered tools that have been seized by some, a bit like bones, carefully torn apart and cleaned - they have torn from a body, a living body that they pretend to ignore...

「Omitted」

Again, this is a formulation that has emerged from more than a year of reflection and investigation. But surely, it was something that was already perceived 'somewhere', from the first years after my departure. Apart from Deligne's work on the absolute values of the Frobenius eigenvalues (the "prestige question", as I have understood lately...) - when I met from time to time one of my acquaintances of yesteryear, with whom I had worked on the same sites*[chantiers]*, and I asked him "so what...?", it was always the same eloquent gesture, with arms in the air as if asking for grace... Clearly, everyone was busy with more important things than I cared about - and clearly also, while everyone was busy looking busy and important, not much was being done. The essential has disappeared - a **unity** that gave meaning to the partial tasks, and a **warmth** too, I think. What was left was a scattering of tasks detached from the whole, each one in his own corner covering his little hoard, or making it grow as best he could.

Even though I would have liked to defend myself against it, it pained me of course to see that everything had come to a halt; to hear no more about motives, or topos, or the six operations, or De Rham's coefficients, or those of Hodge, or the "mysterious functor" which was to link together, in the same range, around De Rham's coefficients, the l-adic coefficients for all prime numbers, nor the crystals (except to learn that they are always at the same place), or the "standard conjectures" and others that I had worked out and which, obviously, represented crucial questions. Even the vast foundational work begun with the Elements of Algebraic Geometry (with Dieudonné's tireless assistance), which it would have been quite sufficient to continue on the momentum already gained, was left to one side: everyone was content to settle into the walls and furniture that someone else had patiently assembled, mounted and bricked up. With the worker gone, it would not have occurred to anyone to roll up their sleeves in turn and put their hand to the trowel, to build the many buildings that remained to be constructed, <u>houses</u>, good to live in, for themselves and for all ...

I could not help but, again, follow up with fully conscious images, which emerged and came up by virtue of reflection. But there is no doubt in my mind that these images must have been present in one form or another, in the deep layers of my being. I must have already felt the insidious reality of a <u>Burial</u> of my work as well as of my person, which suddenly imposed itself on me, with unquestionable force and with this very name, "The Burial", on April of last year. On the conscious level, however, I would hardly have thought of taking offence or even of grieving. After all, being 'close' to the past or not is a matter for the individual, and what one chooses to do with his time. If what had once seemed to motivate or inspire him no longer inspired him, that was his business, not mine. If the same thing seemed to happen, as perfect whole, to all my ex-students without exception, that still was the business of each of them separately, and I had other things to worry about than finding out what meaning it might have, that’s all! As for the things I have left behind, and to which a deep and unknown link continues to connect me - even though they are visibly abandoned on these desolate building sites, I do know well that they are not of the kind that fear the 'ravages of time' or the fluctuations of fashions. If they have not yet become part of the common heritage (as it had seemed to me a while ago), they can not fail to take root there sooner or later, in ten years or in a hundred, it doesn't really matter...

4. A burial wind... 「Omitted」

5. The voyage 「Omitted」

June 1985

6. The shadow side - or creation and contempt

「Omitted」

When I set about this somewhat unusual "introduction" to a mathematical work in February last year (and the year before that, in June), there were (I think) three main things I wanted to say. First of all, I wanted to explain my intentions in returning to a mathematical activity, and the spirit in which I had written this first volume of “Pursuing Stacks" (which I had just declared finished), and also the spirit in which I intended to pursue an even wider journey of mathematical exploration and discovery, with the "Reflections". It would no longer be a matter of presenting meticulous, pinpointed foundations for some new mathematical universe in the making. They would be "logbooks" rather, in which the work would continue from day to day, without hiding anything from it, and as it **really** goes on, with its failures and mistakes, its insistent backward steps and also its sudden leaps forward - a work drawn forward irresistibly day after day (and notwithstanding the innumerable incidents and unforeseen events), as if by an invisible thread - by some elusive, tenacious and sure vision. It is often a groping work, especially in those "sensitive moments" when some as yet nameless and faceless intuition emerges, barely perceptible; or at the start of some new voyage, at the call and pursuit of some first ideas and intuitions, often elusive and reluctant to let themselves be grasped in the meshes of language, when it is often precisely the language adequate to grasp them delicately that is still lacking. It is such a language, above all else, that it is then a question of condensing out of an apparent nothingness of impalpable mists. What is still only sensed, before it is even glimpsed and even less "seen" and touched with the finger, gradually decants from the imponderable, emerges from its cloak of shadows and mists to take on form and flesh and weight...

It is this part of the work, which looks shabby if not (often) lousy, that is also the most delicate and essential part - the part where something <u>new</u> really emerges, through intense attention, solicitude and respect for this fragile, infinitely delicate thing about to be born. This is the creative part of all - that of conception and slow gestation in the warm darkness of the womb, from the invisible original double gamete, becoming a formless embryo and transforming itself over days and months, through obscure and intense work, invisible and formless, into a new being in flesh and blood.

This is also the "dark", "yin" or "feminine" part of the discovery work. The complementary aspect, the "light" or "yang" or "masculine" part, is more akin to working with a hammer or a sledgehammer, a sharp chisel or a wedge of good, hardened steel (tools that are already ready for use, and of proven effectiveness...) Both aspects have their raison d'être and their function, in inseparable symbiosis with each other - or to put it better, they are the bride and groom of the indissoluble couple of the two original cosmic forces, whose unceasingly renewed embrace constantly resurrects the obscure creative labours of conception, gestation and birth - the birth of the child, of the new thing.

「Omitted」

7. Respect and fortitude 「Omitted」

8. "My close ones' - or connivance

It is not my intention in this letter to review all the "strong moments" (or all the "sensitive moments") in the writing of Harvest and Sowing, or in any of its stages([1]). Suffice it to say that there were, in this work, four clearly marked stages or four "breaths" - like the breaths of a respiration, or like the successive waves in a train of waves that arose, I don't know how, from these vast, silent, immobile and moving masses, without limits and without a name, from an unknown and bottomless sea that is "me", or rather, from a sea infinitely larger and deeper than this "me" that it carries and nourishes. These "breaths" or "waves" have materialised in the four parts of Harvest and Sowing now written. Each wave came without my having called for it or planned it in the least, and at no time could I have said where it would take me or when it would end. And when it had ended and a new wave had already taken its place, for a while I thought I was still at the end of a momentum (which would also be, at the end of the ends, the end of Harvest and Sowing!), whereas I was already being lifted and carried towards another breath of the same vast movement. It is only in retrospect that this becomes clear and that a structure is unequivocally revealed in what had been experienced as an act and a movement.

「Omitted」

9. The skimming 「Omitted」

10. Four waves in one movement 「Omitted」

11. Movement and structure

「Omitted」

[1] You will find a short retrospective-balance of the first three parts of Harvest and Sowing in the two groups of notes “The fruits of the evening" (n°s 179-182) and "Discovery of a past" (n° 183-186).

This form is the reflection and expression of a certain spirit, which I have tried to ‘convey*[passer]*' in the preceding pages. Compared to my past publications, if there is a new quality that appears in Harvest and Sowing, and also in “Pursuing Stacks" from which it comes, it is undoubtedly spontaneity. Of course, there are some common threads and major questions that give coherence and unity to the whole reflection. However, this is done on a day-to-day basis, without any pre-established 'programme' or 'plan', without ever fixing in advance 'what had to be demonstrated'. My aim is not to demonstrate, but to discover, to penetrate further into an unknown substance, to condense what is still only sensed, suspected or glimpsed. I can say, without any exaggeration really, that in this work, there is not a single day or night of reflection that has taken place in the field of the “foreseen*[prévu]*”, in terms of the ideas, images, associations that were present when I sat down in front of the white sheet of paper, to stubbornly pursue a tenacious "thread", or to take up another that has just appeared. Each time, what appears in the reflection is different from what I would have been able to predict, if I had ventured to try to describe in advance as best I could what I thought I saw before me. More often than not, the reflection goes down paths that were entirely unforeseen at the outset, leading to new, equally unforeseen landscapes. But even if it was sticked to a more or less foreseen itinerary, what the journey reveals to me as the hours go by differs very much from the image I had of it when I set off, like a real landscape, with its play of fresh shadows and warm light, its delicate and changing perspective according to the steps of the hiker, and these innumerable sounds and nameless perfumes carried by a breeze that makes the grasses dance and the forests sing... - such a living, elusive landscape differs from a postcard, however beautiful and successful, however “correct*[juste]*" it may be.

「Omitted」

12. Spontaneity and rigour

Spontaneity and rigour are the two "shadow" and "light" sides of the same undivided quality. It is only from their marriage that this particular quality of a text, or of a being, is born, which we can try to evoke by an expression like "quality of truth". If in my past publications, spontaneity has been (if not absent, at least) at a minimum, I do not think that its late blossoming in me has meant that rigour has become less so. Rather, the full presence of its yin companion gives rigour a new dimension, a new fruitfulness.

This rigour is exercised with regard to itself, ensuring that the delicate "sorting" it must carry out in the multitude of what passes through the field of consciousness, in order to constantly decant the significant or the essential from the incidental or the accessory, does not thicken and freeze into automorphisms of censorship and complacency. Only curiosity, the thirst for knowledge within us, awakens and stimulates such vigilance without heaviness, such liveliness, in contrast to the immense, omnipresent inertia of the "(so-called) natural slopes", carved out by ready-made ideas, expressions of our fears and our conditioning*[conditionnements]*.

And this same rigour, this same vigilant attention is also directed towards spontaneity as well as towards that which takes on the aspects of it, in order to distinguish, again, from those 'slopes' which are all natural, of course, to distinguish them from that which truly springs from the deepest layers of being, from the original impulse of knowledge and action, carrying us to the encounter with the world.

In terms of writing, rigour manifests itself in a constant concern to define as precisely and as faithfully as possible, with the help of language, the thoughts, feelings, perceptions, images, intuitions...; it is a question of expressing, without being satisfied with a vague or approximate term where the thing to be expressed has clearly defined contours, nor with a term of artificial precision (therefore just as distorting) to express a thing that remains surrounded by the mists of what is still only sensed. When we try to capture it as it is in the moment, and only at the moment, the unknown thing reveals its true nature to us, and even in the full light of day perhaps, if it is made for the day and our desire incites it to strip itself of its veils of shadow and mist. Our role is not to pretend to describe and fix what we do not know and what escapes us, but to humbly, passionately, take note of the unknown and the mystery that surrounds us on all sides.

This means that the role of writing is not to record the results of a search, but the very process of the search - the labours of love and the works of our loves with Our Mother the World, the Unknown, who unceasingly calls us into Her to know Her again in Her inexhaustible Body, wherever in Her the mysterious ways of desire take us.

In order to make this process possible, the backtracking, which nuances, clarifies, deepens and sometimes corrects the "first draft*[jet]*" of the writing, or even a second or third, is part of the discovery process itself. They form an essential part of the text and give it its full meaning. This is why the "notes" (or "annotations") placed at the end of Fatuité et Renouvellement, and referred to here and there in the course of the fifty "sections" that constitute the "first draft" of the text, are an inseparable and essential part of it. I strongly advise you to refer to them as you go along, and at least at the end of each section where there are one or more references to such "notes". The same applies to the footnotes in the other parts of Harvest and Sowing, or to the references, in such and such a "note" (constituting here the "main text"), to such and such a later note, which then acts as a "return" on it, or as an annotation. This, along with my advice not to separate yourself from the table of contents, is the main recommendation I have for your reading.

「Omitted」

Alexandre Grothendieck

Postscript epilogue - or context and preconditions for a debate

February 1986

13. The bottle spectrograph 「Omitted」

14. Three feet in one plate

「Omitted」

c) There is a second way in which many of my colleagues and former friends remain confused. It is the "sorry, but we are not specialists in this field - don't ask us to take cognisance of facts, which (providentially...) pass over our heads...".

I affirm, on the contrary, that to be aware of the main facts, you don't need to be a "specialist" (sorry my turn!), nor even to know your multiplication table or the Pythagoras theorem. You don't even need to have read "Le Cid" or the Fables of La Fontaine. A normally developed ten-year-old is just as capable as the most renowned specialist (or even better than him...)([1]).

Let me illustrate this point with just one example, the "first one" from the Burial([2]). You don't need to know the ins and outs of the multifaceted and very delicate mathematical notion of "motive", nor do you even need to have a school certificate, in order to take cognisance of the following facts, and to make a judgment on their subject.

1) Between 1963 and 1969 I introduced the notion of "motive", and I developed around this notion a " philosophy " and a " theory ", which remained partially conjectural. Rightly or wrongly (it doesn't matter here), I consider the theory of motives to be the most profound thing I have contributed to the mathematics of my time. The importance and depth of "motivic yoga" is no longer disputed by anyone (after ten years of almost complete silence on the subject, right after my departure from the mathematical scene).

2) In the first and only book (published in 1981), essentially devoted to the theory of motives (and where this name, introduced by me, appears in the title of the book), the one and only passage that could make the reader suspect that my modest person is closely or remotely linked to some theory that could resemble the one developed at length in this book, is on page 261. This passage (of two and a half lines) consists in explaining to the reader that the theory developed there has nothing to do with that of a man called Grothendieck (a theory mentioned there for the first and last time, without any other reference or clarification).

3) There is a famous conjecture, the so-called "Hodge conjecture" (whatever it is about), whose validity would imply that the so-called "other" theory of motives developed in the brilliant volume is <u>identical</u> to (a very special case of) the one I had developed, in plain sight of everyone, almost twenty years before.

「Omitted」

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[1] Of course, I did not write Harvest and Sowing for the ten-year-old child, to address him I would have chosen a language that would be familiar to him.

[2] It was the first 'big operation' of Burial that I discovered, on a certain April 19, 1984, when the name 'The Burial' also imposed itself on me. See on this subject the two notes written on the same day, "Recollection of a dream - or the birth of the motives", and "The Burial - or the New Father" (ReS II, n°s 51, 52). The full reference of the book to be discussed is also included.

15. The gangrene - or the spirit of the age (1) 「Omitted」

16. Honorable fine - or the spirit of the age (2) 「Omitted」

Introduction (I): The five-leaf clover

1. Dream and fulfilment

Three years ago in July, I had an unusual dream. If I say "unusual", it is an impression that came to me only afterwards, when I thought about it on waking. The dream itself came to me as the most natural, obvious thing in the world, without any fanfare - so much so that when I woke up, I almost didn't pay any attention to it, just pushed it into the dustbin and moved on to the "order of the day". Since the day before, I had been thinking about my relationship with mathematics. It was the first time in my life that I had bothered to look into it - and even then, if I did, it was because I was really almost forced to! There were such strange, not to say violent, things that had happened in the previous months and years, sort of explosions of mathematical passion bursting into my life out of nowhere, that it was really no longer possible not to look at what was going on.

The dream I am talking about had no scenario or action of any kind. It consisted of a single image, motionless, but at the same time very much alive. It was the head of a person, seen in profile. It was seen looking from right to left. It was a middle-aged man, beardless, with wild hair forming a halo of strength*[force]* around his head. The impression that emerged from this head was that of a youthful, joyful strength, which seemed to spring from the supple and vigorous arch of the neck (which one would guess more than one could see). The expression on his face was more that of a mischievous boy, delighted at some trick he had just played or was thinking of playing, than that of a mature man, or of one who had grown up*[pris de l'assiette]*, mature or not. Above all, he exuded an intense joy of life, restrained*[contenue]*, bursting into play...

There was no second person present, an "I" who looked at or contemplated this other person, whose head was all that could be seen. But there was an intense perception of this head, of what was emerging from it. Nor was there anyone to feel the impressions, to comment on them, to name them, or to stick a name to the perceived person, to designate him as "such a one". There was only this very living thing, this man's head, and an equally living, intense perception of this thing.

When I woke up, without any deliberate intention, I remembered the dreams of the past night, the vision of this man's head did not stand out in number with any particular intensity, it did not push itself forward to shout or whisper to me: it is me you have to look at! When this dream appeared in the field of my quick glance at the dreams of the night, in the warm tranquillity of the bed, I had of course this reflex of the awakened mind to put a name on what had been seen. I didn't have to look for it, I only had to ask the question to know at once that this man's head which had been there in the dream was none other than mine.

It's not bad, I thought, it's necessary to do it anyway, to see oneself in a dream like that, as if it were someone else! This dream came about as if, while walking around and by the greatest of chance, I had come across a four-leaf clover, or even a five-leaf clover, to marvel at it for a few moments as I should, and to continue on my way as if nothing had happened.

At least that's how it almost happened. Fortunately, as has happened to me many times in situations like this, I nevertheless, out of a sense of conscience, wrote down this little "not bad" incident in black and white, starting a reflection that was supposed to continue the work of the previous day. Then, one thing leading to another, the reflection of that day was limited to immersing myself in the meaning of this unpretentious dream, this unique image, and the message about myself that it brought me.

This is not the place to dwell on what this one-day meditation taught me and brought me. Or rather, what this dream taught me and brought me, once I had put myself in a state of attention, of listening, which allowed me to take in what it had to tell me. The first immediate fruit of the dream and of this listening was a sudden influx of new energy. This energy carried the long-lasting meditation that continued in the following months, against stubborn inner resistances, which I had to dismantle one by one by patient and obstinate work.

In the five years since I began to pay attention to some of the dreams that came to me, this was the first "messenger dream" which did not present itself in the now recognisable appearance of such a dream, with impressive scenic means, and an exceptional, sometimes overwhelming intensity of vision. This one was very "cool", with nothing to force the attention, even discretion - it was, take it or leave it, no fuss...

A few weeks earlier I had had a messenger dream in the old style, on the dramatic and even savage diapason, which put a sudden and immediate end to a long period of mathematical frenzy. The only apparent similarity between the two dreams is that in neither was there an observer. In a parable of lapidary force, this dream showed something that was going on in my life at the time, which I did not bother to pay attention to - something that I took great care to ignore, in fact. It was this dream that made me realise the urgency of a work of reflection, in which I engaged a few weeks later, and which then continued for almost six months. I have the opportunity to talk about it a little in the last part of this reflection-testimony "Harvest and Sowing", which opens this volume and gives it its name([1]).

I began this introduction with an evocation of this other dream, this image-vision of myself ("Traumgesicht meiner selbst*[dream face of myself -Trans.]*" as I called it in my memos*[notes]* in German), because in the last few weeks the thought of this dream came back to me more than once, as the meditation "on a mathematician's past" was coming to an end. In fact, in retrospect, the three years that have passed since the dream appear to me as years of decantation and maturation, towards a fulfilment of its simple and clear message. The dream showed me "as I am". It was also clear that in my waking life I was not fully who the dream showed me to be - weights and stiffnesses from afar often got in the way of my being fully and simply myself. During these years, when the thought of this dream rarely came back to me nevertheless, this dream must have taken effect*[agir]* in a certain way. It was by no means a kind of model or ideal to which I would have strived to be like, but a quiet reminder of a joyful simplicity that "was me", manifested in many ways, and was called upon to free itself from what continued to weigh upon it and to blossom fully. This dream was a delicate and vigorous link at the same time, between a present still weighed down by many weights stemming from the past, and a very close "tomorrow" that this present contains in germ, a "tomorrow" which is me from the present on, and which has surely always been in me...

Surely, if in these last weeks this rarely evoked dream has been present again, it is because at a certain level, which is not the level of the mind that probes and analyses, I must have "known" that the work I was doing and

[1] See in particular section 43, "The troublemaker boss - or the pressure cooker".

bringing to its end, a work that took up again and deepened that other work of three years ago, was a new step towards the fulfilment of the message about myself that it brought me.

This is now the main meaning for me of Harvest and Sowing, of this intense work of almost two months. Only now that it has been completed do I realise how important it was that I did it. In the course of this work I have had many moments of joy, often mischievous, joking, exuberant joy. And there have also been moments of sadness, and moments when I have relived frustrations or sorrows that have affected me painfully in recent years - but there has not been a single moment of bitterness. I leave this job with the complete satisfaction of one who knows he has completed a job. There is nothing, no matter how "small", that I have avoided, or that I would have liked to say but did not say, which at this moment would leave in me the residue of dissatisfaction, of regret, however "small" it might be.

In writing this testimony, it was clear to me that it will not please everyone. It is even possible that I have found a way to displease everyone without exception. However, that was not my intention at all, nor was it my intention to displease anyone. My aim was simply to look at the simple and important things, the everyday things, of my past (and sometimes of my present as well) as a mathematician, to discover at last (better late than never!) and without the shadow of a doubt or reservation, what they were and what they are; and, on the way, to say in simple words what I saw.

2. The spirit of a voyage

This reflection, which eventually became "Harvest and Sowing", had begun as an "introduction" to the first volume (in the process of being completed) of "Pursuing Stacks", the first mathematical work that I have been aiming to publish since 1970. I had written the first few pages at a slack[creux] moment, in June last year, and I resumed this reflection less than two months ago, from the point where I had left it. I realised that there was quite a lot of things to look at and to say, so I expected a relatively extensive introduction, thirty or forty pages long. Then, for the next two months or so, until now when I am writing this new introduction to what was originally an introduction, I thought every day that this would be the day I finish this work, or that it would be the next day or the day after at worst. When after a few weeks I started to approach the hundred-page mark*[cap]*, the introduction was promoted to "introductory chapter". After a few more weeks, when the dimensions of this "chapter" were far exceeding those of the other chapters of the volume in preparation (all finished at the time of writing these lines, except for the last one), I finally understood that its place was not in a maths book, that this reflection and this testimony would definitely be cramped there. Their true place was in a separate volume, which will be volume 1 of these "Mathematical Reflections" that I intend to continue in the years to come, following the momentum of Pursuing Stacks.

I would not say that Harvest and Sowing, this first volume in the series of Mathematical Reflections (to be followed by the two or three volumes of Pursuing Stacks, to begin with) is an 'introductory' volume to the Reflections. Rather, I see this first volume as the foundation of what is to come, or rather, as the one that gives the background note, the spirit in which I undertake this new voyage, which I intend to pursue in the years to come, and which will take me I cannot say where.

To conclude these details about the main part of the present volume, here are a few indications of a practical nature. The reader will not be surprised to find in the text of Harvest and Sowing occasional references to the 'present volume' - by which I mean the first volume (History of Models) of Pursuing Stacks, whose introduction I believe I am still writing. I did not want to "correct" these passages, wishing above all to preserve the spontaneity of the text, and its authenticity as a testimony not only to the distant past, but also to the very moment I am writing.

It is also for the same reason that my retouching of the first draft of the text was limited to correcting stylistic clumsiness or a sometimes confused expression that hindered the understanding of what I wanted to express. These retouches sometimes led me to a clearer or more refined understanding than when I wrote the first draft. The slightly substantial modifications to the text, to nuance, clarify, complete or (sometimes) correct it, are the subject of some fifty numbered notes, grouped together at the end of the reflexion, and which constitute more than a quarter of the text([1]). I refer to them by symbols such as (1) etc... Among these notes, I have singled out twenty or so that seemed to me of comparable importance (in length or substance) to any of the fifty "sections" or "paragraphs" into which the reflection was spontaneously organised. These longer notes have been included in the table of contents, after the list of the fifty sections. Not surprisingly, for some of the longer notes, there was a need to add one or more notes to the note. These are then included following the note, with the same type of references, except for the shorter notes, which then appear on the same page as "footnotes", with references such as (*) or (**).

I took great pleasure in naming each of the sections of the text, as well as each of the more substantial notes - not to mention the fact that, later on, this proved to be indispensable for my navigation. It goes without saying that these names were found after the fact, whereas at the beginning of a section or a note that was a little long I would not have been able to tell for any of them what the essential substance would be. The same applies a fortiori to the names (such as "Work and discovery", etc...) by which I have designated the eight parts I to VIII into which I have afterwards grouped the fifty sections that make up the text.

For the content of these eight parts, I will limit myself to very brief comments. The first two parts I (Work and discovery) and II (The Dream and the Dreamer) contain elements of a reflection on mathematical work, and on the work of discovery in general. I myself am involved in a much more episodic and much less direct way than in the following parts. It is these parts in particular that have the quality of testimony and meditation. Parts III to VI are mainly a reflection and testimony on my past as a mathematician "in the mathematical world", between 1948 and 1970. The motivation behind this meditation was above all the desire to understand this past, in an effort to understand and take on a present that is sometimes disappointing*[décevants]* or confusing*[déroutants]*. Parts VII (The Child is having fun) and VIII (The solitary adventure) are more concerned with the evolution of my relationship to mathematics from 1970 to the present, that is, since I left the "world of mathematicians" and never returned. In particular, I examine the motivations, and the forces and circumstances, that led me (to my own surprise) to resume 'public' mathematical activity (writing and publishing Mathematical Reflections), after an interruption of over thirteen years.

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[1] (28 May) This is the text of the first part of Harvest and Sowing, "Fatuity and Renewal". The second part was not written at the time of writing.

3. Compass and luggage

I should say a few words about the other two texts which, together with Harvest and Sowing, make up the present volume of the same name.

"Sketch of a Programme" gives a sketch of the main themes of mathematical reflection that I have pursued over the last ten years. I intend to develop at least some of them in the years to come, in a series of informal reflections that I have already had occasion to mention, the "Mathematical Reflections". This sketch is the textual reproduction of a report I wrote last January to support my application for a research post at the CNRS. I have included it in the present volume, because it is clear that this programme is far beyond the possibilities of my modest self, even if I were given another hundred years to live, and I choose to use them to follow the themes in question as far as I can.

"Thematic Sketch" was written in 1972 on the occasion of another application (for a professorship at the Collège de France). It contains a sketch, by themes, of what I then considered to be my main mathematical contributions. This text is affected by the circumstances in which it was written, at a time when my interest in mathematics was marginal, to say the least. So this sketch is little better than a dry and methodical enumeration (but which fortunately does not aim to be exhaustive...). It does not seem to be carried by a vision or by the breath of a desire - as if these things that I review like a matter of conscience (and these were indeed my dispositions) had never been touched by a living vision, nor by a passion to bring them to light when they were still only sensed behind their veils of mist and shadow...

If I have decided to include this uninspiring report here, I am afraid, it is mainly to silence*[clore le bec]* (assuming that it is possible) to certain high-flying colleagues and to a certain fashion, who since my departure from the world that was common to us, have been looking down on what they kindly call "grothendieckeries". This, it seems, is synonymous with bombinage*[bombinage]* on things too trivial for a serious mathematician of good taste to consent to waste precious time on them. Perhaps this indigestible "digest" will seem more "serious" to them! As for the texts from my pen, which are driven by a vision and a passion, they are not for those whom a fashion sustains and justifies in complacency, making them insensitive to the things that enchant me. If I write for others than for myself, it is for those who do not consider their time and their person too precious to pursue without ever getting tired of the obvious things that no one deigns to see, and to rejoice in the intimate beauty of each of the things discovered, distinguishing it from any other that was known to us in its own beauty.

If I wanted to situate the three texts that make up the present volume in relation to each other, and the role of each in the voyage on which I have embarked with the Mathematical Reflections, I could say that the reflection-testimony Harvest and Sowing reflects and describes the spirit in which I am undertaking this voyage and which gives it its meaning. Sketch of a Programme describes my sources of inspiration, which set a direction, if not a destination, for this voyage into the unknown, somewhat like a compass, or a strong Ariadne's thread. Finally, Thematic Sketch quickly reviews a baggage, acquired in my past as a mathematician before 1970, at least part of which will be useful and desirable in this or that stage of the voyage (just as my reflexes of cohomological and toposic algebra are indispensable to me now in Pursuing Stacks). And the order in which these three texts follow each other, as well as their respective lengths, reflect well (without any deliberate intention on my part) the importance and the weight I give them in this voyage, whose first stage is approaching its end.

4. A voyage in pursuit of obvious things...

I should say a few more detailed words about this voyage undertaken a little over a year ago, the Mathematical Reflections. I will explain myself in some detail, in the first eight sections of Harvest and Sowing (i.e. in parts I and II of the reflection) about the <u>spirit</u> in which I undertook this voyage, and which, I think, is apparent already in the present first volume, as well as in the one that will follow (the History of Models, which is volume 1 of Pursuing Stacks), which is in the process of being completed. It therefore seems unnecessary to expand on this subject in this introduction.

I certainly cannot predict what this voyage undertaken will be, something I will discover as it continues... I do not have an itinerary planned even in outline at present, and I doubt that one will come out soon. As I said earlier, the main themes that will undoubtedly inspire my reflection are more or less sketched out in "Sketch of a Programme", the "compass-text". Among these themes, there is also the main theme of Pursuing Stacks, that is to say, the "stacks", which I hope to cover (and leave it at that) in the course of this year again, in two or perhaps three volumes. About this theme I write in the Sketch: "... it is a bit like a debt that I would pay to a scientific past where, for about fifteen years (between 1955 and 1970), the development of cohomological tools was the constant Leitmotiv in my work on the foundations of algebraic geometry". It is therefore the one that is, among the planned themes, most strongly rooted in my scientific "past". It is also the one that has remained present as a regret throughout these past fifteen years, as the most glaring gap of all, perhaps, in the work that I had left to do when I left the mathematical scene, and that none of my students or friends of yesteryear bothered to fill. For more details on this work in progress, the interested reader may refer to the relevant section in Sketch of a Programme, or to the introduction (the real one this time!) to the first volume, in progress, of Pursuing Stacks.

Another legacy of my scientific past that is particularly dear to me, is the notion of <u>motive</u>, which is still waiting to emerge from the night, where it has been kept for a good fifteen years since it first made its appearance. It is not excluded that I will end up working on the foundations that are necessary here, if no one better placed than me (by virtue of a younger age, as well as by the tools and knowledge at his disposal) decides to do so in the next few years.

I take this opportunity to point out that the fortune (or rather, misfortune...) of the notion of motive, and of some others among those I have drawn to light and which, among all of them, seem to me (in potential) the most fruitful, are the subject of a retrospective reflection of nearly twenty pages, forming the longest (and one of the very last) of the "notes" to Harvest and Sowing([1]). I have subsequently subdivided this note into two parts ("My orphans" and "Refusal of a legacy - or the price of a contradiction"), in addition to the three "sub-notes"

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[1] This double note (no. 46, 47) and its sub-notes have been included in the second part 'The Burial' of Harvest and Sowing, which is a direct continuation.

that follow it([1]). These five consecutive notes are the only part of Harvest and Sowing where mathematical notions are evoked other than by passing allusions. These notions become the occasion to illustrate certain contradictions within the world of mathematicians, which themselves reflect contradictions in the people themselves. At one point I considered separating this sprawling note from the text from which it comes, and attaching it to Thematic Sketch. This would have had the advantage of putting the latter into perspective, and breathing some life into a text that looks a bit too much like a catalogue. However, I have refrained from doing so, in order to preserve the authenticity of a testimony of which this meganote, whether I like it or not, is indeed a part.

To what is said in Harvest and Sowing about the dispositions in which I approach the "Reflections", I would like to add one thing here, on which I have already expressed myself in one of the notes ("The snobbery of the young - or the defenders of purity"), when I write: "My ambition as a mathematician throughout my life, or rather my joy and passion, has always been to discover the obvious things, and this is my only ambition also in the present work" (Pursuing Stacks). This is my only ambition also in this new voyage that I have been pursuing for a year with the Reflections. It has been no different in these Harvests and Sowings which (for my readers at least, if there are any) open up this voyage.

5. A welcome debt

I would like to conclude this introduction with a few words about the two dedications in this volume "Harvest and Sowing".

The dedication "to those who were my students, to whom I gave the best of myself - and also the worst" has been present in me at least since last summer, and especially when I wrote the first four sections of what was still supposed to be an introduction to a mathematical work. That is to say, I knew well, in fact for some years already, that there was a "worst" to be examined - and now was the time or never! (But I had no idea that this "worst" would eventually lead me through a meditation of almost two hundred pages).

On the other hand, the dedication "to those who were my elders" appeared only along the way, just like the name of this reflection (which also became the name of a volume). It revealed to me the important role they played in my life as a mathematician, a role whose effects are still alive today. This will no doubt become clear enough in the pages that follow, so that there is no need to dwell on it here. These 'elders', in (approximate) order of appearance in my life when I was twenty, are Henri Cartan, Claude Chevalley, André Weil, Jean-Pierre Serre, Laurant Schwartz, Jean Dieudonné, Roger Godement, Jean Delsarte. The ignorant newcomer that I was was welcomed with benevolence by each of them, and thereafter many of them gave me lasting friendship and affection. I must also mention here Jean Leray, whose kind reception during my first contact with the 'world of mathematicians' (in 1948/49) was also a precious encouragement. My reflection revealed a debt of gratitude to each of these men "from another world and another destiny". This debt is by no means a burden. Its discovery came as a joy, and made me lighter.

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[1] These are sub-notes no. 48, 49, 50 (note no. 48' was added later).

End March 1984

Introduction (II): An act of respect

(4 May - ... June)

6. The Burial

An unforeseen event re-launched a reflection that had been concluded. It started a cascade of discoveries large and small over the past weeks, gradually revealing a situation that had remained unclear and sharpening its contours. In particular, this led me to enter in detail and in depth into events and situations that had previously only been mentioned in passing or by allusion. As a result, the "fifteen-page retrospective reflection" on the vicissitudes of a work, mentioned earlier (Introduction, 4), has taken on unexpected dimensions, expanding by some two hundred additional pages.

By force of circumstance and by the inner logic of a reflection, I have been led along the way to involve others as much as myself. The one who is involved more than anyone else (apart from myself) is a man to whom I have been friends for almost twenty years. I have written of him (euphemistically([1])) that he had 'been a bit of a pupil', in the very early years of this affectionate friendship rooted in a common passion, and for a long time and deep in my heart I saw in him a sort of 'legitimate heir' of what I thought I could contribute in mathematics, beyond a published work that remained fragmentary. Many people will already have recognised him: he is Pierre Deligne.

I make no apology for making public with these notes, among others, a personal reflection on a personal relationship, and for involving him in this way without having consulted him. I think it is important, and healthy for everyone, that a situation that has remained hidden and confused for a long time is finally brought to light and examined. In doing so, I am providing a testimony, admittedly subjective, which does not claim to exhaust a delicate and complex situation, nor to be free of errors. Its first merit (like that of my past publications, or those on which I am now working) is that it exists, available to those who may be interested. My concern has been neither to convince, nor to shelter myself from error or doubt behind the only things that are said to be "obvious". My concern is to be true, by saying things as I see or feel them, in each moment, as a means to deepen them and to understand.

The name "The Burial", for the whole of the notes relating to "The weight of a past", imposed itself with increasing force in the course of the reflection([2]). I play the role of the anticipated deceased, in the funereal

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[1] On the meaning of this 'euphemistically', see the note 'Being apart', no. 67'.

[2] Towards the end of this reflection, another name came up, expressing another striking aspect of a certain picture that had gradually been unfolding in my eyes over the past five weeks. It is the name of a tale, which I shall return to in its place: "The robe of the Chinese Emperor"...

company of the few (much younger) mathematicians whose work takes place after my 'departure' in 1970 and bears the mark of my influence, by a certain style and by a certain approach to mathematics. Foremost among these is my friend Zoghman Mebkhout, who had the heavy privilege of having to face all the handicaps of being treated as a 'student of Grothendieck after 1970', without having had the advantage of contact with me and my encouragement and advice, whereas he was only a 'student' of my work through my writings. This was at a time when (in the world he haunts) I was already such a "deceased" figure that for a long time the very idea of a meeting apparently did not arise, and an ongoing relationship (both personal and mathematical) was finally established only last year.

This did not prevent Mebkhout, against a tyrannical fashion and the disdain of his elders (who were my students) and in almost complete isolation, from doing an original and profound work, through an unexpected synthesis of the ideas of the Sato school and mine. This work provides a new insight into the cohomology of analytic and algebraic varieties, and holds the promise of a far-reaching renewal in our understanding of this cohomology. No doubt this renewal would have been accomplished by now and for years, if Mebkhout had found among those designated for this purpose the warm welcome and unreserved support that they once received from me. At least, since October 1980 his ideas and works have provided the inspiration and the technical means for a spectacular restart of the cohomological theory of algebraic varieties, finally emerging (apart from Deligne's results around Weil's conjectures) from a long period of stagnation.

Unbelievably, and yet truly, his ideas and results have been used by "everyone" for nearly four years (just like mine), while his name remains carefully ignored and silenced by the very people who know his work first hand and use it in an essential way in their work. I do not know of any other time when mathematics has been so disgraced, when some of the most influential or prestigious of its adherents set an example, with general indifference, of disregard for the most universally accepted rule of the ethics of the mathematical profession.

I see four men, mathematicians of brilliant means, who had and have the right to be along with me to receive the honours of this burial by silence and by disdain. And I see in each of them the bite of contempt on the beautiful passion that had animated him.

Apart from these, I see above all two men, both in the limelight in the mathematical public square, who officiate at funerals in large company and who at the same time (in a more hidden sense) are buried and with their own hands, together with those whom they deliberately bury. I have already named one of them. The other is also a former student and friend, Jean-Louis Verdier. After my 'departure' in 1970, contact between him and me was not maintained, apart from a few hasty meetings on a professional level. This is why he is probably only mentioned in this reflection through certain acts in his professional life, while the possible motivations for these acts, in terms of his relationship with me, are not examined and, moreover, entirely escape me.

If there is one pressing question that has imposed itself on me throughout the past years, which has been a profound motivation for Harvest and Sowing and which has also followed me throughout this reflection, it is that of the part that I have played in the advent of a certain spirit and certain manners that make possible disgraces such as the one I have mentioned, in a world that was mine and with which I had identified myself for more than twenty years of my life as a mathematician. The reflection made me discover that by certain attitudes of fatuity in me, expressed by a tacit disdain of colleagues with modest means, and by an indulgence*[complaisance]* to myself and to such mathematicians with brilliant means, I was not foreign to this spirit which I see spreading today among those I had loved, and among those also to whom I taught a trade I loved; those whom I have disliked and taught badly and who today set the tone (if not the law) in this world which was dear to me and which I have left.

I feel a wind of smugness, cynicism and contempt blowing. "It blows without regard to "merit" or "demerit", burning with its breath the humble vocations as well as the most beautiful passions...". I have understood that this wind is the prolific harvests of blind and careless*[insouciantes]* sowings that I have helped to sow. And if its breath comes back on me and on what I had entrusted to other hands, and on those whom I love today and who dared to claim themselves or merely to be inspired by me, this is a <u>return of things</u> of which I have no reason to complain, and which has much to teach me.

7. Funeral Arrangement

Under the name "The Burial", I have therefore grouped together in the table of contents the imposing parade of the main "notes" relating to this seemingly anodyne section "The weight of a past" (s. 50), thus giving full meaning to the name that had immediately imposed itself on me for this final section of the "first draft" of Harvest and Sowing.

In this long procession of notes with multiple relatives, those that were added during the past four weeks (notes (51) to (97)([1])) stand out as the only ones dated (from 19 April to 24 May)([2]). It seemed most natural to give them in the chronological order in which they follow one another in the reflection([3]), rather than in any other so-called "logical" order, or in the order in which references to these notes appear in earlier notes. In order to find this latter (by no means linear) order of filiation between participating notes, I have followed (in the table of contents) the number of each one by that of the note (among those preceding it) where it is first referred to([4]), or

[1] Note no. 104 of 12 May 1984 should also be added. The notes no. 98 and its following (with the exception of the previous note no. 104) constitute the "third wind*[souffle]*" of the reflection, from 22 September 1984. They are also dated.

[2] In a series of consecutive notes written on the same day, only the first is dated. The other undated notes are no. 44' to 50 (forming processions I, II, III). Notes no. 46, 47, 50 are from 30 or 31 March, notes no. 44', 48, 48', 49 from the first half of April, finally note no. 44" is dated (10 May).

[3] I have sometimes reversed this chronological order slightly, in favour of a 'logical' order, when I felt that this did not distort the overall impression of the process of reflection. As the only exceptions, however, I would like to mention eleven notes (whose number is preceded by the sign !) taken from footnotes subsequent to a note and which have grown to prohibitive dimensions, each of which I have placed following the note to which they refer (except for note no. 98, which refers to no. 47).

[4] When the reference to a note (such as (46)) is in the section "The weight of a past" itself, it is the number (50) of the latter, <u>placed in parentheses</u>, that is placed after that of the note, such as 46 (50).

(otherwise) by the number of the one of which it constitutes an immediate continuation[1]. (This last relationship is indicated in the text itself by a reference symbol placed at the end of the first note, such as (!47) placed at the end of the last line of note (46), which refers to note (47) that it continues). Finally, certain details of a somewhat technical nature to a note are grouped together at the end of the note in sub-notes numbered by indices consecutive to the number of the original note - as in sub-notes (46.1) to (46.9) of note (46) 'My orphans'.

In order to give some structure to the overall sequencing of the Burial, and to enable one to recognise the multitude of notes that crowd together, I deemed it appropriate for this purpose to include in the procession some seriously suggestive subtitles, each one preceding and leading a long or short procession of consecutive notes linked by a common theme.

Thus I had the pleasure of seeing, assembled one by one, in a long solemn parade*[venant]* to honour my funeral, ten[2] processions - some humble, others imposing, some contrite and others secretly jubilant, as cannot be otherwise on such an occasion. So here we go: the <u>posthumous pupil</u> (whom everyone makes a point of ignoring), the <u>orphans</u> (freshly exhumed for the occasion), <u>Fashion</u> and its <u>illustrious Men</u> (I am well-deserved for that), the <u>motives</u> (the last born and last exhumed of all my orphans), <u>my friend Pierre</u> modestly leading the largest of the processions, followed closely by the <u>Unanimous Agreement</u> of (silently) concertante notes by the <u>Colloquium</u> (called "Pervers") in full (standing out from the posthumous pupil, alias the Unknown Pupil, by interposed funeral processions bearing flowers and wreaths); finally, to close the imposing procession with dignity, here comes the <u>Pupil</u> (by no means posthumous and even less unknown) alias <u>the Boss</u>, followed by the busy troop of <u>my pupils</u> (equipped with shovels and ropes) and finally the <u>Funeral Van</u> (displaying four beautiful oak coffins solidly screwed together, not counting the Gravedigger)... ten processions finally in full force (it was about time), slowly making their way to the <u>Funeral Ceremony</u>.

The highlight of the Ceremony is the Funeral Eulogy, served with perfect skill by none other than my friend Pierre himself, presiding over the funeral in response to the wishes of all and to the general satisfaction. The Ceremony ends in a final and definitive (one hopes at least) De Profundis, sung as a sincere thanksgiving by the late deceased himself, who, unbeknownst to all, survived his impressive funeral and even learned from it*[pris de la graine]*, to his <u>complete satisfaction</u> - a satisfaction which forms the final and ultimate note of the memorable Funeral.

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[1] The number of a note which is an immediate continuation of a preceding note (whose numbers then follow each other) is preceded by the sign * in the table of contents. Thus *47, 46 indicates that note no. 47 is an immediate continuation of note no. 46 (not the one immediately preceding it, which is note no. 46.9).

Finally, I have underlined in the t. of m. the numbers of the notes which are not followed by another number, that is to say those which represent a "new beginning" of the reflection, not inserted in a specific place of the reflection already made.

[2] (29 September) In fact, there are in the end <u>twelve</u> processions, including the Funeral Van (X), and "The (still undead) deceased" (XI) who has just snuck into the procession again in extremis...

8. The end of a secret

In the course of this ultimate stage (I hope) of the reflection, it became clear to me that it would be useful to include, as an "Appendix" to the present volume 1 of Mathematical Reflections, two other texts of a mathematical nature, in addition to the three mentioned above([1]).

The first is the reproduction of a commented report in two parts, which I made in 1968 and 1969 on the work of P. Deligne (some of which remains unpublished to this day), corresponding to a mathematical activity at the IHES during the three years 1965/67/68.

The other text is a sketch of a "form of the six variances", gathering the features common to a duality formalism (inspired by Poincaré's duality and Serre's duality) that I had worked out between 1956 and 1963, a formalism which proved to have a "universal" character for all the cohomological duality situations encountered to date. This formalism seems to have fallen into disuse with my departure from the mathematical scene, to the extent that to my knowledge no one (apart from myself) has yet bothered to write down even the list of fundamental operations, the fundamental canonical isomorphisms to which they give rise, and the essential compatibilities between them.

This sketch of a coherent form will be for me the first obvious step towards this "vast overall picture of the dream of motives", which for more than fifteen years "has been waiting for the bold mathematician who will be willing to paint it". In all likelihood, this mathematician will be none other than myself. It is indeed high time that what was born and entrusted in intimacy nearly twenty years ago, not to remain the privilege of one but to be available to all, finally emerges from the night of secrecy, and is born once again into the full light of day.

It is true that only one, apart from myself, had an intimate knowledge of this "yoga of motives", having learned it from myself in the days and years before I left. Of all the mathematical things I had been privileged to discover and bring to light, this reality of motives still appears to me as the most fascinating, the most charged

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[1] In addition, I am thinking of adding to Thematic Sketch (see "Compass and Baggage", Introduction, 3) a "commentary" giving some details about my contributions to the "themes" that are summarily reviewed there, and about the influences that took part in the genesis of the main key-ideas in my mathematical work. The retrospective of the last six weeks has already revealed (to my own surprise) a role of "detonator" of Serre, for the start of most of these ideas, as well as for some of the "great tasks" I had set myself, between 1955 and 1970.

Finally, as another text of a mathematical nature (in the common sense), and the only one that appears (incidentally) in the non-technical text "Harvest and Sowing", I point out sub-note no. 87.1 to the note "The massacre" (no. 87), where I explain with the care it deserves a "discrete" (conjectural) variant of the familiar Riemann-Roch-Grothendieck theorem in the coherent context. This conjecture appeared (among a number of others) in the closing exposé of the SGA 5 seminar of 1965/66, an exposé of which no trace remains (any more than many others) in the volume published eleven years later under the name SGA 5. The vicissitudes of this crucial seminar in the hands of some of my students, and the links of these with a certain 'SGA 4 1/2 operation', are gradually revealed in the course of the reflection pursued in notes no. 63''', 67, 67', 68, 68', 84, 85, 85', 86, 87, 88.

As another note giving quite extensive mathematical commentaries, on the appropriateness*[opportunité]* of a common (as far as possible) "toposic" framework for the known cases where a "six operations" duality formalism is available, I also point out sub-note no. 81.2 to the note "Thesis on credit and all risks insurance", no. 81.

with mystery - at the very heart of the deep identity between "geometry" and "arithmetic". And the 'yoga of motives' to which this long-ignored reality has led me is perhaps the most powerful instrument of discovery I have unleashed in this early*[première]* period of my mathematical life.

But it is also true that this reality, and the 'yoga' that tries to capture it as closely as possible, had not been kept secret by me. Absorbed by the imperative tasks of writing the foundations (which everyone has since been happy to use as they are in their daily work), I did not take the few months necessary to write down a vast sketch of this yoga of the motives, and thus make it available to everyone. However, in the years preceding my unexpected departure, I did not fail to speak about it at random and to anyone who wanted to hear about it, starting with my students, who (apart from one of them) have forgotten it, just as everyone else has forgotten it. If I spoke about it, it was not to place "inventions" that would bear my name, but to draw attention to a reality that manifests itself at every step, as soon as one is interested in the cohomology of algebraic varieties and in particular, in their "arithmetic" properties and in the relations between them of the different cohomological theories known to date. This reality is as tangible as that of the "infinitesimally small", perceived long before the appearance of the rigorous language which allowed to apprehend it in a perfect way and to "establish it". And to apprehend the reality of motives, we are today by no means short of a flexible and adequate language, nor of a consummate experience in the construction of mathematical theories, which our predecessors lacked.

If what I once shouted from the rooftops has fallen on deaf ears - and if the disdainful silence of one has echoed the silence and lethargy of all those who pretend to be interested in cohomology (and yet have eyes and hands just like me...), I cannot hold the one person responsible who chose to keep the "profit" of what I had entrusted to him for the interest of all. It is clear that our era, whose unbridled scientific productivity rivals that invested in armaments or consumer goods, is very far from the "bold dynamism" of our seventeenth-century predecessors, who "did not beat about the bush" to develop a calculation of the infinitesimally small, without worrying about whether this calculation was "conjectural" or not; nor did they wait for some prestigious man among them to give them the green light, in order to grasp what everyone could see with their own eyes and feel at first hand.

9. The Stage and the Actors

By its own internal structure and its particular theme, "The Burial" (which now forms more than half of the text of Harvest and Sowing) is to a large extent and logically independent of the long reflection that precedes it. This is, however, a superficial independence. For me, this reflection, around a "burial" gradually emerging from the mists of the unspoken and the prescient, is inseparable from the one that preceded it, from which it is derived and which gives it its full meaning. Started as a quick glance 'in passing' at the vicissitudes of a work I had lost sight of a little (a lot), it became, without having planned or sought it out, a meditation on an important relationship in my life, leading me in turn to a reflection on the fate of this work in the hands of 'those who were my pupils'. To separate this reflection from the one from which it spontaneously sprang seems to me a way of reducing it to a simple “picture of manners" (or even, to a settlement of accounts in the mathematical "beautiful world”).

It is true that, if one insists on it, the same reduction to a "picture of manners" can be made for the entire Harvest and Sowing. Certainly, the manners which prevail at a given era and in a given milieu, and which contribute to shaping the lives of the men who are part of it, are important and deserve to be described. However it will be clear to a careful reader of Harvest and Sowing that my purpose is not to describe manners, in other words a certain stage*[scène]*, changing with time and from one place to another, on which our actions take place. This stage to a large extent defines and delimits the means available to various forces within us, allowing them to express themselves. While the stage and the means it provides (and the 'rules of the game' it imposes) vary infinitely, the nature of the deep forces within us that (at the collective level) shape the stages and that (at the level of the individual) express themselves on them, seems to be the same from one milieu or culture to another, and from one time to another. If there is one thing in my life, apart from mathematics and the love of a woman, that I have felt the mystery and attraction of (late in life, admittedly), it is the hidden nature of those few forces that have the power to make us act, for "better" or "worse", to bury and to create.

10. An act of respect

This reflection, which eventually took the name "The Burial", had begun as an act of respect. A respect for things that I had discovered, that I saw condense and take shape in a void, whose taste and vigour I was the first to know and to which I gave a name, to express both the knowing I had of them, and my respect. To these things I gave the best of myself. They have been nourished by the strength that rests in me, they have grown[poussé] and flourished[épanouies], like multiple and vigorous branches springing from the same living trunk with multiple vigorous roots. These are living and present things, not inventions that can be made or not made - things closely interwoven in a living unity that is made of each of them and gives each its place and meaning, an origin and an end. I had left them long ago and without any worry or regret, for I knew that what I left was healthy and strong and did not need me to grow and flourish again and multiply, following its own nature. It was not a bag of money I was leaving behind, which could be stolen, nor a pile of tools, which could rust or rot.

And yet, as the years went by, when I thought I was far away from a world I had left behind, here and there, even in my retreat, I heard puffs of insidious disdain and discreet derision, pointing to those things I knew to be strong and beautiful, which had their own place and their own unique function that no other thing could ever fulfil. I felt them to be orphans in a hostile world, a world sick with the disease of contempt, lashing out[acharnant] at that which is unarmoured. It was in this mood that this reflection began, as an act of respect for these things and thus for myself - as a reminder of a deep connection between these things and me: for anyone who takes pleasure in showing disdain for any of these things that have been nurtured by my love, it is me, and everything that has come from me, that he is taking pleasure in disdaining.

And the same is true of anyone who, knowing first-hand the link between me and something he has learned from no one but me, pretends to consider this link negligible or to ignore it, or to claim (even tacitly and by omission) for himself or for someone else a fictitious 'paternity'. I clearly see in this an act of contempt for a thing born of the worker as well as for the obscure and delicate work that allowed this thing to be born, and for the worker, and above all (in a more hidden and essential way) for himself.

If my 'return to maths' were to serve only to remind me of this link and to arouse in me this act of respect in front of all - in front of those who affect to disdain and in front of the indifferent witnesses - this return will not have been in vain.

It is true that I had really lost touch with the written and unwritten (or at least unpublished) work I had left behind. As I began this reflection, I could see the branches quite clearly, but I didn't really remember that they were part of the same tree. Strangely enough, it took the gradual unveiling of the picture of a devastation of what I had left behind, for me to regain a sense of the living unity of what was thus devastated and dispersed. One took away a few coins and the other a tool or two to avail himself of or even to use them - but the unity that makes the life and the true strength of what I had left, it has escaped each and everyone. I know of one person, however, who has felt this unity and this strength deeply, and who still feels it in his heart today, and who likes to disperse the strength that is in him in order to destroy this unity that he has felt in others through his work. It is in this living unity that the beauty and creative virtue of the work lies. Despite the devastation, I find them intact as if I had just left them - except that I have matured and now see them with new eyes.

If anything, however, is devastated and mutilated, and defused of its original power, it is in those who forget the power that lies within themselves and those who imagine that they are devastating a thing at their mercy, when they are only cutting themselves off from the creative virtue of that which is at their disposal as it is at the disposal of all, but by no means at their mercy or in the power of anyone.

Thus this reflection, and through it this unexpected "return", will also have made me reconnect with a forgotten beauty. It is to have fully felt this beauty that gives all the meaning to this act of respect which is awkwardly expressed in the note "My orphans"([1]), and which I have just reiterated in full awareness of the matter*[cause]* here.

[1] This note (no. 46) is chronologically the first of all those that appear in The Burial.